Gravitation and the Planets

• **Goals:**
  – How did we discover we orbit the Sun.
  – What laws define the orbits of the planets.
  – What are the laws of Newtonian Mechanics (Gravity).

• **Geocentric Cosmogonies**
  – **Does the Sun orbit the Earth?**
    
    *see Figures 4-2, 4-3*
  
    • Greeks believed that the stars were fixed on a rotating celestial sphere.
  
    • The Sun and moon move in the opposite direction (west-east).
  
    • The planets ("wanderers") move west-east ("direct") and east-west ("retrograde").
  
    • Ptolemaic system invoked planets rotating in epicycles (small circles) with the center orbiting the earth (a deferent).
  
    • Mars: retrograde every 22.5 months.
Gravity/Planets

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Diagram showing the concept of epicycle and deferent.

- Epicycle
- Planet
- Deferent

Earth
Planet moves rapidly eastward along epicycle

Epicycle moves slowly eastward along deferent

As seen from Earth, planet moves eastward (direct motion)
Planet moves rapidly westward along epicycle

Epicycle moves slowly eastward along deferent

As seen from Earth, planet moves westward (retrograde motion)
• **Heliocentric Cosmogonies**
  – **No absolute frame of reference**
    • From an object orbiting a planet the planet appears to be orbiting it!
    • E.g. on a train as it moves away from the station - the station appears move.
  – **Retrograde motion**
    [see Figure 4-4]
    • Each planet orbits the Sun at a different period - we “over take” the other planets.
    • Retrograde motion comes from our motion.
  – **Copernican Model (1500s)**
    [see Figure 4-6]
    • The radius of a planets orbit and its position along that orbit defines its place on the sky.
    • Planets between Earth and Sun - inferior
    • Planets beyond the Earths orbit - superior
New Observations of the Planets

• **Tycho Brahe**
  
  – **Parallax**
    
    • If an observer moves then foreground objects change relative to the background.
    
    • Closing each eye changes the image you see (e.g. a pen in front of your face).
    
    • The movement of the foreground object is greater the closer it is to you.
    
    • The rotation and orbit of the earth can cause this effect.
  
  – **New observations**
    
    • High precision (1 arcmin) positions of the stars.
    
    • Could not measure parallax - believed the Earth was stationary (parallax of nearby stars is < 1 arcsec).

• **Johannes Kepler**
  
  – **New model for the orbit of the planets**
    
    • Motion of planets are described by ellipses
Apparent position now

Nearby object

Apparent position 12 hours later

Now

12 hours later
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Gravity/Planets

Diagram showing the Earth's orbit around the Sun, with a nearby star at the focus. The diagram includes labels for the Earth (July) and Earth (January), with the Sun at the center and a distance labeled 1 AU. The nearby star is marked with an arrow indicating its position relative to the Sun and Earth.
– **Ellipses**
  
  • Circle has one focus - ellipse has 2 foci.
  • Shape of the ellipse is defined by the semi-major \((a)\) and semi-minor \((b)\) axes.
  • Circle is a special case of an ellipse.
  • Elliptical orbits match the data precisely.

– **Kepler’s First law**
  
  • Orbit of a planet about the Sun is an ellipse with the Sun at one focus.
  • The length of the semi-major axis is the average distance from the planet to the Sun.
  • Ellipse shape given by eccentricity \((e)\)
    \[ e = \frac{\sqrt{a^2 - b^2}}{a} \]
  
  - Venus \(e=0.007\)
  - Pluto \(e=0.248\)
    - (circle) \(0 < e < 1\) (line)
  • Perihelion (closest to Sun): Planet moves rapidly.
  • Aphelion (furthest from Sun): Planet moves less rapidly.
Major axis

Minor axis

Focus

Focus
Kepler’s Second Law

Figure 4-11

• A line joining a planet and the Sun sweeps out the equal areas in equal amounts of time.
• A circle is an idealized case of this.

Kepler’s Third Law

• The square of the sidereal period of a planet is directly proportional to the cube of the semi-major axis of the orbit.

\[ P^2 = a^3 \]

\[ P \text{ (Sidereal Period) measured in years} \]
\[ a \text{ (semi-major axis) measured in AU} \]

• Larger the orbit, longer the period and slower the average speed.
• Elliptical orbit with variable speed removes the need for retrograde corrections.

Kepler discovered these relations empirically
A fundamental result
• **Galileo Galilei**
  
  – **Verified Keplers results with new observations**
    - Orbits of the moons around Jupiter.
    - Phases of Venus.
    - Rings of Saturn.
    - Sunspots.
    - Mountains on the moon.
  
  – **Each observation helped improve the validity of the Copernican model**

  **Figures 4-14, 4-15**

  - Variable size of Venus (and its relation to its phases) cannot be explained by the Ptolemaic model.
  - Period of the moons of Jupiter depended on their semi-major axis.
  - A paradigm shift.
<table>
<thead>
<tr>
<th>Date</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>15 Jan.</td>
<td></td>
</tr>
<tr>
<td>26 Jan.</td>
<td></td>
</tr>
<tr>
<td>31 Jan.</td>
<td></td>
</tr>
<tr>
<td>2 Feb.</td>
<td></td>
</tr>
<tr>
<td>3 Feb.</td>
<td></td>
</tr>
<tr>
<td>9 Feb.</td>
<td></td>
</tr>
<tr>
<td>13 Feb.</td>
<td></td>
</tr>
<tr>
<td>16 Feb.</td>
<td></td>
</tr>
<tr>
<td>17 Feb.</td>
<td></td>
</tr>
<tr>
<td>18 Feb.</td>
<td></td>
</tr>
<tr>
<td>19 Feb.</td>
<td></td>
</tr>
<tr>
<td>20 Feb.</td>
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</tr>
<tr>
<td>21 Feb.</td>
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</tr>
<tr>
<td>22 Feb.</td>
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</tr>
<tr>
<td>23 Feb.</td>
<td></td>
</tr>
<tr>
<td>24 Feb.</td>
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<td>26 Feb.</td>
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<td>27 Feb.</td>
<td></td>
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<tr>
<td>28 Feb.</td>
<td></td>
</tr>
<tr>
<td>29 Feb.</td>
<td></td>
</tr>
<tr>
<td>1 Mar.</td>
<td></td>
</tr>
</tbody>
</table>
– **Periods of the planets**

- Period is related to the radius of the orbit.
- Synodic period (S) - time between 2 identical configurations relative to earth.
- Sidereal period (P, E) - time for a true orbit.

\[
\text{Earth Orbits at } \frac{360^\circ}{E} \Rightarrow \text{ in one Synodic Period } \frac{360^\circ}{S} \\
\text{Planet Orbits at } \frac{360^\circ}{P} \Rightarrow \text{ in one Synodic Period } \frac{360^\circ}{S}
\]

**Inferior Planet: Planet “laps” Earth**

\[
\frac{360}{P} S = \frac{360}{E} S + 360 \quad \Rightarrow \quad \frac{1}{P} = \frac{1}{E} + \frac{1}{S}
\]

- Jupiter: $S=398.9$ days $= 1.092$ yrs
  $P$ (Sidereal) $= 11.87$ yrs

**Superior Planet: Planet “lags” Earth**

\[
\frac{360}{P} S = \frac{360}{E} S - 360 \quad \Rightarrow \quad \frac{1}{P} = \frac{1}{E} - \frac{1}{S}
\]
<table>
<thead>
<tr>
<th>Planet</th>
<th>Synodic period</th>
<th>Sidereal period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>116 days</td>
<td>88 days</td>
</tr>
<tr>
<td>Venus</td>
<td>584 days</td>
<td>225 days</td>
</tr>
<tr>
<td>Earth</td>
<td>—</td>
<td>1.0 year</td>
</tr>
<tr>
<td>Mars</td>
<td>780 days</td>
<td>1.9 years</td>
</tr>
<tr>
<td>Jupiter</td>
<td>399 days</td>
<td>11.9 years</td>
</tr>
<tr>
<td>Saturn</td>
<td>378 days</td>
<td>29.5 years</td>
</tr>
<tr>
<td>Uranus</td>
<td>370 days</td>
<td>84.0 years</td>
</tr>
<tr>
<td>Neptune</td>
<td>368 days</td>
<td>164.8 years</td>
</tr>
<tr>
<td>Pluto</td>
<td>367 days</td>
<td>248.5 years</td>
</tr>
</tbody>
</table>
### Table 4-2: Average Distances of the Planets from the Sun

<table>
<thead>
<tr>
<th>Planet</th>
<th>Copernican value (AU*)</th>
<th>Modern value (AU)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>0.38</td>
<td>0.39</td>
</tr>
<tr>
<td>Venus</td>
<td>0.72</td>
<td>0.72</td>
</tr>
<tr>
<td>Earth</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Mars</td>
<td>1.52</td>
<td>1.52</td>
</tr>
<tr>
<td>Jupiter</td>
<td>5.22</td>
<td>5.20</td>
</tr>
<tr>
<td>Saturn</td>
<td>9.07</td>
<td>9.54</td>
</tr>
<tr>
<td>Uranus</td>
<td>—</td>
<td>19.19</td>
</tr>
<tr>
<td>Neptune</td>
<td>—</td>
<td>30.06</td>
</tr>
<tr>
<td>Pluto</td>
<td>—</td>
<td>39.53</td>
</tr>
</tbody>
</table>

*1 AU = 1 astronomical unit = average distance from the Earth to the Sun.*
## Table 4-3: A Demonstration of Kepler’s Third Law

<table>
<thead>
<tr>
<th>Planet</th>
<th>Sidereal period $P$ (years)</th>
<th>Semimajor axis $a$ (AU)</th>
<th>$P^2$</th>
<th>$a^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>0.24</td>
<td>0.39</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>Venus</td>
<td>0.61</td>
<td>0.72</td>
<td>0.37</td>
<td>0.37</td>
</tr>
<tr>
<td>Earth</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Mars</td>
<td>1.88</td>
<td>1.52</td>
<td>3.53</td>
<td>3.51</td>
</tr>
<tr>
<td>Jupiter</td>
<td>11.86</td>
<td>5.20</td>
<td>140.7</td>
<td>140.6</td>
</tr>
<tr>
<td>Saturn</td>
<td>29.46</td>
<td>9.54</td>
<td>867.9</td>
<td>868.3</td>
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<tr>
<td>Uranus</td>
<td>84.01</td>
<td>19.19</td>
<td>7,058</td>
<td>7,067</td>
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<tr>
<td>Neptune</td>
<td>64.79</td>
<td>30.06</td>
<td>27,160</td>
<td>27,160</td>
</tr>
<tr>
<td>Pluto</td>
<td>248.54</td>
<td>39.53</td>
<td>61,770</td>
<td>61,770</td>
</tr>
</tbody>
</table>
Newtonian Mechanics

• **Newton’s first Law (law of inertia)**
  – A body remains at rest or moves in a straight line at constant speed unless acted upon by a net **outside** force.
    • Net force = combined effect of all outside forces (vector sum).
    • With no gravity planets would “fly off”. Gravity maintains the orbits.
    • Law is related to inertia (we stay in the same state unless acted upon).

• **Newton’s Second Law**
  – Acceleration of an object is directly proportional to the net outside force.

\[
F = ma
\]

F: force
m: mass
a: acceleration
• **Newton's Second Law**
  
  – **Acceleration: rate of change of velocity**
    
    • Acceleration can change the speed or direction of an object.
    • Planets are accelerated by Gravity (changing their direction).
    • Acceleration occurs in the direction of the force.
    • Force must overcome inertia to accelerate.
  
  – **Example**
    
    • Earth's surface gravity is 9.8 m/s².
    • A 50 kg person jumps from a plane.
      
      \[
      F = ma \\
      = 50 \times 9.8 = 490 \text{ Newtons} = 110 \text{ pounds}
      \]
    • On the moon (\( g = 1.666 \text{ m/s}^2 \)).
      
      \[
      F = 50 \times 1.666 = 83.3 \text{ Newtons} = 18.7
      \]
    • Weight (Force) „ Mass (Inertia)
• Newtons Third Law
  – When one body exerts a force on another body the second body exerts an equal and opposite force on the first body.
    • Equal and opposite forces act on different bodies.
  – Example:
    • A shotgun - pellets and the gun receive the same force. The mass of the pellets is much smaller so they accelerate faster.
    • mass of shot = 1oz = 0.025 kg.
    • mass of person = 50 kg.
    \[ F_{\text{shot}} = F_{\text{person}} \]
    \[ a_{\text{shot}} = \frac{m_{\text{person}}}{m_{\text{shot}}} a_{\text{person}} \]
    \[ a_{\text{shot}} = 2000 a_{\text{person}} \]
    • Sun exerts a force on the planets and they exert an equal and opposite force of the Sun (Earth is 1/300,000 mass of Sun: acceleration of the Sun is small).
Strong pull required

Ball moves at a high speed in a small circle
b  Ball moves at a low speed in a large circle
• **Gravitation and the Planets**
  - **Gravity shapes the orbits**
    - The pull of gravity makes planets “fall” towards the Sun (same as the pull of the earth on an apple).
    - The smaller the semi-major axis - faster rotation, requires more force.

• **Newton’s Law of Gravity**
  - Two bodies attract each other with a force that is directly proportional to the mass of each body and inversely proportional to the square of the distance between them.

\[
F = \frac{G m_1 m_2}{r^2}
\]

- \(F\): Force
- \(m_1\): mass of object 1
- \(m_2\): mass of object 2
- \(r\): distance between objects
- \(G\): Gravitational constant \((6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2})\)

  - Double distance: quarter force.
  - Double mass: double force.
Example:

- Sun-Earth and Earth-Sun.
  \[ M_{\text{sun}} = 1.99 \times 10^{30} \text{ kg} \]
  \[ M_{\text{earth}} = 5.98 \times 10^{24} \text{ kg} \]
  \[ r = 1 \text{ AU} = 1.5 \times 10^{11} \text{ m} \]
  \[ F_{\text{S.E}} = 6.67 \times 10^{-11} \frac{5.98 \times 10^{24} \times 1.99 \times 10^{30}}{(1.5 \times 10^{11})^2} \]
  \[ F_{\text{S.E}} = 3.53 \times 10^{22} \text{ N} \]

Why do we orbit not fall into the Sun?

Figure 4-20

- Tangential motion stops the Earth falling directly towards the Sun.
- If speed is right curvature of the path matches curvature of Sun (circular orbit).
- If the speed is greater or slower (a little) the orbit becomes elliptical.
- If speed is much greater we escape from orbit (hyperbolic or parabolic curve).
- The centripetal acceleration \( a_r \) required to keep the ball in orbit with velocity \( v \) is
  - Using \( F=ma \) implies that the orbital velocity is given by
    \[ v = \left( \frac{GM}{r} \right)^{1/2} \]
    \[ a_r = \frac{v^2}{r} \]
– **Surface Gravity (g).**
  
  • A measure of the acceleration of an object at the planet (or stars) surface.
  
  • From Newton's Law of Gravity the acceleration is independent of the mass of the object, hence given special symbol g.
  
  • M= Mass of planet, R= Radius of planet.
  
  • \( g = \frac{GM}{R^2} \)
    
    – \( g = 9.8 \) m/s\(^2\) for Earth.
    
    – g for the Moon is a factor of 6 smaller than g for the Earth.

– **Conservation of Energy**

  • The total energy of an enclosed system is constant. Energy may be converted from one form to another but not created or destroyed.
  
  • Energy --- the ability to do work.
  
  • Kinetic energy --- energy of motion = \( E_k = \frac{1}{2} m v^2 \)
  
  • Potential Energy --- \( E_p = -\frac{GMm}{r} \)
– Conservation of Momentum

• Total momentum of particles in a system remains unchanged.

• Linear momentum
  – \[ \text{Linear momentum} = m \cdot \vec{v} \] (vector).
  – Linear momentum is constant unless acted on by an EXTERNAL force.

• Angular momentum (L).
  – The angular momentum (L) of an object moving with velocity \( V \) with respect to an axis at distance \( r \) is
  – \[ L = m \cdot r \times \vec{v} \] (cross product)
  – Kepler's 2nd law is simply conservation of Angular momentum.
--- Escape velocity ---

- A measure of the velocity required by an object to escape the gravitational pull of the planet (or star). The escape velocity, $V_{\text{esc}}$, is defined such that it is the minimum speed an object needs so that it will never fall back to the planets surface.

- At infinity $E_k + E_p = 0$, since object just makes it to infinity.

- As object is fired from the Earth
  - $E_k = \frac{1}{2} m v^2$
  - $E_p = -\frac{G M m}{r}$

- Since energy is conserved we have $E_k + E_p = 0$ hence
  - $\frac{1}{2} m v^2 = \frac{G M m}{r}$

- Thus (for a spherical Earth)
  - $V_{\text{esc}} = (\frac{2G M}{R})^{1/2}$

- where $M$ is the Mass of the planet, and $R$ its radius.
• **Newton's form of Kepler’s law**

\[ P^2 = \frac{4\pi^2}{G(m_1 + m_2)}a^3 \]

- \( a \): semi-major axis (average distance).
- **Many applications**
  - Measure the masses of binary stars.

• **Progress of Science**
  - **Kepler: New observations**
  - **Newton: New model/theory**
    - Theory predicted Halleys comet’s return and was used in the discovery of Neptune.
    - Cornerstone of modern physics.
This person is at low tide

This person is at high tide

Moon

Earth

Oceans

This person is at high tide

This person is at low tide

Spring tide

Neap tide

Moon

Sun

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Gravity/Planets