Spectral Line Measurements in Exceptionally Low SNR Achieved by virtue of the KLT (Karhunen-Loève Transform)

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A little-known tool for spectral line measurements is the KLT (acronym for Karhunen-Loève Transform). This mathematical algorithm is superior to the classical FFT in many regards:
1) The KLT can filter signals out of the background noise over both wide and narrow bands. That is in sharp contrast to the FFT that rigorously applies to narrow-band signals only.
2) The KLT can be applied to random functions that are non-stationary in time, i.e. whose autocorrelation is a function of the two independent variables t1 and t2 separately. Again, this is a sheer advantage of the KLT over the FFT, inasmuch as the FFT rigorously applies to stationary processes only, i.e. processes whose autocorrelation is a function of the absolute value of the difference of t1 and t2 only.
3) It can detect signals embedded in noise to unbelievably small values of the Signal-to-Noise Ratio (SNR), like 10^-4 or so. This particular feature of the KLT is studied in detail in this paper. As a practical application, we show the case of a 43 GHz line emission produced by SiO and detected experimentally in R Cassiopeae by virtue of the KLT already available at the IRA-INAF Noto radiotelescope in Sicily, Italy. Many more similar examples could be given as well.
An excellent filtering algorithm as the KLT, however, comes with a cost that one must be ready to pay for: its computational burden is much higher than for the FFT. In fact, it can be shown that no ***fast*** KLT transform can possibly exist and, for an autocorrelation matrix of size N, the calculations must be of the order of N^2, rather than N*log(N). Nevertheless, for moderate values of N (in the hundreds) the KLT dominates over the FFT, as we show in this paper with mathematical details.