**Motion of cometary dust**

Marco Fulle  
*INAF – Osservatorio Astronomico di Trieste*

**Summary.** On time–scales of days to months, the motion of cometary dust is mainly affected by solar radiation pressure, which determines dust dynamics according to its cross–section. Within this scenario, dust motion builds–up structures named dust tails. Tail photometry, depending on the dust cross section too, allows models to infer the best available outputs describing fundamental dust parameters: mass loss rate, ejection velocity from the coma, and size distribution. Only models taking into account all these parameters, each strictly linked to all others, can provide self–consistent estimates of each of them. All available tail models find that comets release dust with its mass dominated by the largest ejected boulders. A much more unexpected result is however possible: also coma brightness may be dominated by meter–sized boulders. This result, if confirmed by future observations, would require deep revisions of most dust coma models, based on the common assumption that coma light is coming from grains of sizes close to the observation wavelength.

1. **HISTORICAL OVERVIEW**

While ices sublimate on the nucleus surface, the dust embedded in it is released and dragged out by the gas expanding in the coma. The dust motion then depends on the following boundary conditions: the 3D nucleus topography and the complex 3D gas–dust interaction close to the nucleus surface. Both these conditions were never available to modelers. Dust coma shapes heavily depend on the details of the boundary conditions, and coma models cannot disentangle the effect of dust parameters on coma shapes from those due to the unknown boundary conditions. On the contrary, dust tails are usually structureless, suggesting that here dust has lost memory of all the details of the boundary conditions. Moreover, solar radiation pressure acts like a mass spectrometer, putting dust of different mass in different space positions: this allows models to disentangle how dust parameters affect tail shapes, making tail models powerful tools to describe dust in comets.

A dust tail is a broad structure which originates from the comet head and can reach lengths from $10^4$ to $10^8$ km in the most spectacular cases (Fig. 1). When we consider the comet orbital path and the straight line going from the Sun to the comet nucleus (named radius vector), we divide the comet orbital
plane in four sectors (Fig. 2). All cometary tails lie in the sector out of the comet orbit and behind the comet nucleus. When we project these four sectors of the comet orbital plane onto the sky, they may be strongly deformed by the observation perspective when the Earth is close to the comet orbital plane. In these cases, the sector where tails are possible may be a complete sky halfplane: the appearance of a perspective anti–tail is then possible, which, although seen to roughly point to the Sun, always is external to the comet orbit. A few years ago, a particular kind of anti–tails was discovered (Panseccchi et al. 1987), which lie in the sector opposite to that where most tails are possible. These are much shorter than usual tails, not exceeding $10^6$ km in length (Sec. 7.1).

The first tail model in sufficient agreement with observations was developed by F. W. Bessel (1784 – 1846) and then refined by F. A. Bredichin (1831 – 1904), who introduced the definitions of synchrones and syndynes (Fig. 2). They assumed that a repulsive force, inversely proportional to the squared Sun–comet distance, would act on the material composing the tails. According to this hypothesis, the tail particle is subjected to a total force equal to the solar gravity times a factor $\mu$ characterizing the tail particle. When this repulsive force is equal to the solar gravity, i.e. when $\mu = 0$, the tail particle moves according to an uniform straight motion. When $\mu < 0$, the tail particle moves on a hyperbola with the convexity directed towards the Sun. When $0 < \mu < 1$, the tail particle behaves as the Sun were lighter, so that its motion is slower than that of the comet nucleus and on an orbit external with respect to the comet nucleus. This model explained why cometary tails always lie in the comet orbital plane sector defined above. S. Arrhenius (1859 – 1927) proposed the solar radiation pressure as a candidate for the Bessel’s repulsive force. The computations of K. Schwarzschild (1873 – 1916) and P. Debye (1884 – 1966) established that the $\beta = 1 - \mu$ parameter is inversely proportional to the diameter of the dust particle (assuming it to be a sphere) and equal to 1 for dust diameters close to 1 $\mu$m.

Let us consider the comet nucleus moving along its orbit and ejecting dust particles with exactly zero ejection velocity. When we consider particles all ejected at the same time and characterized by any $\mu$ value, we obtain a line family named synchrones, each labeled by its ejection time. Synchrones can be approximated by radial lines, all crossing the comet nucleus. Conversely, if we take into account particles characterized by the same $\mu$ value ejected at any time, we obtain a line family named syndynes, each labeled by its $\mu$ value. Syndynes can be approximated by spirals tangent to the prolonged radius vector at the comet nucleus. In this model, each point of the tail is crossed by only one synchrone and one syndyne. It is so possible to infer the ejection time and the $\mu$ value of each point of the tail. The synchrone–syndyne model predicts that dust tails are 2D structures lying on the comet orbital plane. Tail observations when the Earth crosses the comet orbital plane (Panseccchi et al. 1987) have demonstrated that dust tails are thick, i.e. 3D structures.
2. DUST TAILS: PHOTOMETRIC THEORY

The dust dynamics depend on the $\beta = 1 - \mu$ parameter

$$\beta = 1 - \mu = \frac{3E_{\odot}Q_{pr}}{8\pi cGM_{\odot}\rho_d d} = \frac{C_{pr}Q_{pr}}{\rho_d d} \quad (1)$$

where $E_{\odot}$ is the mean solar radiation, $c$ the light speed, $Q_{pr}$ the scattering efficiency for radiation pressure, $G$ the gravitational constant, $M_{\odot}$ the solar mass, $\rho_d$ the dust bulk density and $d$ the diameter of the dust grains, assumed here to be spheres. Here we define the dimensionless $\rho$ it is convenient to express the DSD in terms of ($\rho_d d$) of the differential dust size distribution (DSD). Since the dust dynamics depend on the $\beta$ the relation between dust mass loss rate and tail brightness depends on the second and third momenta which makes dust tail models a powerful tool to infer the mass loss rate of cometary dust. Eq.(5) shows the relation between the dust mass loss rate $\dot{M}$ and the tail brightness $I$ is independent of $\rho_d$

$$\dot{M} = \frac{\pi}{6} \rho_d < (\rho_d d)^3 >$$

$$I(x, y) = \frac{3}{2} B \int_{-\infty}^{t_o} \int_0^{\infty} K(x, y, t, 1 - \mu) \dot{N}(t) \sigma(t, 1 - \mu) dt d(1 - \mu) \quad (2)$$

which makes dust tail models a powerful tool to infer the mass loss rate of cometary dust. Eq.(5) shows that the relation between dust mass loss rate and tail brightness depends on the second and third momenta of the differential dust size distribution (DSD). Since the dust dynamics depend on the $\beta = 1 - \mu$ quantity, it is convenient to express the DSD in terms of $(\rho_d d)$ weighted by the square of the dust size, providing the average dust cross section. Thus, we define the dimensionless $\beta$ distribution

$$f(t, 1 - \mu)d(1 - \mu) = \frac{(\rho_d d)^2 g(t, \rho_d d) d(\rho_d d)}{\int_0^{\infty} (\rho_d d)^2 g(t, \rho_d d) d(\rho_d d)} \quad (6)$$

$$g(t, \rho_d d) d(\rho_d d) = k(1 - \mu)^2 f(t, 1 - \mu) d(1 - \mu) \quad (7)$$

where $k$ is a dimensionless constant depending on the quantity $B$ defined in Eq.(2). If the DSD $g(t, \rho_d d)$ is a power law versus $(\rho_d d)$ with index $\alpha$, then $f(t, 1 - \mu)$ is a power law versus $(1 - \mu)$ with index $-\alpha - 4$.

Dust tail models provide as output the quantity $F(t, 1 - \mu)$, which has dimensions $m^2 s^{-1}$

$$F(t, 1 - \mu) = \left( \frac{C_{pr}Q_{pr}}{\rho_d} \right)^2 \dot{N}(t) f(t, 1 - \mu) \quad (8)$$
so that the dust mass loss rate given by Eq.(4) becomes

\[ \dot{M}(t) = \frac{\pi}{6} k C_{pr} Q_{pr} \int_{0}^{\infty} \frac{F(t, 1 - \mu)}{1 - \mu} d(1 - \mu) \] (9)

Values of dimensionless quantity \( k \) depend on actual techniques used to observe the dust tail. In case of optical photographic or CCD data, it becomes

\[ k = 4 \frac{A_{p}(\phi)}{D(x, y)} \] (10)

where \( A_{p}(\phi) \) is the geometric albedo times the phase function: an isotropic body diffusing all the received radiation uniformly in all directions has \( A_{p} = \frac{1}{4} \) (Hanner et al. 1981); \( \gamma \) is a radiant expressed in arcsec; \( r \) the sun–comet distance at observation expressed in AU; \( m(x, y) \) the dust tail brightness expressed in mag arcsec\(^{-2} \); \( m_{\odot} \) the Sun magnitude in the bandwidth of tail observations; \( D(x, y) \) the dimensionless tail brightness defined in Eq.(2). CCD data provide much better observational constraints to models, because are independent of plate linearization, shortcoming used on photographic material. Moreover, the much higher efficiency of CCDs allows us to use interferential filters to avoid emissions from ions and gases, which usually pollute wide bandwidths (e.g. Johnson filters) used in astrophotography. However, also CCD data provide dust mass loss rates depending on the poorly known dust albedo. In case of thermal IR or mm data,

\[ k = \frac{4 S_{\nu}(x, y, T)}{\pi D(x, y) B_{\nu}(T)} \] (11)

so that \( k \) is now independent of the dust albedo, depending only on the dust temperature \( T \), usually much better known than albedo. Here, \( B_{\nu}(T) \) is the Planckian at temperature \( T \), and \( S_{\nu}(x, y, T) \) is the observed dust tail in a thermal IR or mm wavelength. If the dust grains have very aspherical shapes, with very different cross sections along each space coordinate (e.g. noodles, long cylinders or flat disks), then it becomes impossible to compute any dust dynamics (Crifo and Rodionov 1999). If dust grains are not spherical but still compact, then the numerical values of \( C_{pr} \) and \( k \) change, while Eqs.(1), (2), (6), (7), (8) and (9) remain identical (where now \( d \) is the mean grain size), as well as Eqs.(3), (4) and (5), after we change only the factors \( \frac{\pi}{4}, \frac{\pi}{6} \) and \( \frac{3}{2} \), respectively. Therefore, thermal IR or mm tail data provide the most reliable information on dust mass loss rate and DSD in comets. In fact, the outputs of IR tail models are completely independent of the dust shape (provided the grains are compact), bulk density and albedo, i.e. the most uncertain parameters of cometary dust. Moreover, parallel IR and optical observations done at the same time can provide unique estimates of the size and time dependence of the dust albedo.

### 3. 2D MODELS

If we assume that dust is ejected from the coma at exactly zero velocity with respect to the nucleus, then the resulting 2D model depends on the quantity \( F \) only defined by Eq.(8). Within this model, the dust
tail is a thin dust layer lying in the comet orbital plane. The dust tail brightness $D$ is simply proportional to $F$ multiplied by the determinant of the Jacobian between the $(x,y)$ frame and the syndyne–synchrone network defined in Sec.1. Then Eq.(5) is directly invertible, easily providing the quantity $F$, from which we can infer the dust mass loss rate by Eq.(9) and the DSD by means of the normalization of $F$ versus $\beta$ and then by Eq.(7). Following such a direct approach, the DSD of short period comets 2P/Encke and 6P/d'Arrest was obtained (Sekanina and Schuster 1978). The DSD power index was always $\alpha < -4$. In this case, both the brightness and the mass mainly depend on the micron-sized grains observed in the tail. If $\alpha > -3$, both the mass and brightness depend on the largest ejected grains. Brightness and mass become decoupled if $-4 < \alpha < -3$, in which case the dust mass depends on the largest ejected grains, while the brightness depends on the micron-sized ones.

Since the observed brightness fixes the number of micron-sized grains observed in the comet if $\alpha < -3$, the DSD index in practice affects only the number of unobserved large grains, which may dominate the ejected mass if $\alpha > -4$ too. Therefore, Sekanina and Schuster (1978) correctly concluded that the index they found ($\alpha = -4.2$) implied the number of ejected large grains was negligible, and this assumption was adopted to design the ESA Giotto mission to 1P/Halley. The impact of Giotto with a grain of 1 g at the flyby implied that the actual probability of such an impact was underestimated by a factor 100 at least. Since the size ratio between 1 g grains and those dominating the brightness of comets 2P/Encke and 6P/d’Arrest was $10^4$, the real $\alpha$ value should have been $\alpha > -3.7$. We should conclude either that the dust population of 1P/Halley is very different from that of 2P/Encke or 6P/d’Arrest, or that the 2D model has intrinsic severe shortcomings.

Both observations when the Earth crosses the comet orbital plane, and hydrodynamic models describing the dust–gas interaction in the coma, point out that the assumption of zero dust ejection velocity is non-physical. We remind that this approximation is useless also for the so–named perspective antitails which are sometimes composed of dust released at very large Sun distances. In this case, the presumably very low dust velocity is balanced by the very long time interval between dust ejection and observation. Since the thickness of the tail is roughly given by such a time interval times the dust velocity, it is always impossible to obtain a dust tail which can be well approximated by a 2D model based on synchrones and syndynes. This is true also for apparently thin antitails observed e.g. in Comet Arend–Roland 1957III: Kimura and Liu (1977) have shown that the correct explanation of these spikes has nothing to do with the 2D syndyne–synchrone model.

After we have shown that the 2D tail model has no physical basis, we have also to show that the outputs of this 2D model are systematically different from those of a correct 3D model applied to the same data. This was done, and in fact the DSD power index resulted $\alpha \approx -3.7$ for both comets 2P/Encke and 6P/D’Arrest (Fulle 1990, Table 1). The fact that the dust mass in 2P/Encke is strongly dominated by
the largest ejected grains was further confirmed applying the same 3D model to independent IR thermal data provided by the ISO probe (Epifani et al. 2001: $\alpha = -3.2$). These results show that 2D models and all related outputs should be avoided in the future, being unable to provide useful constraints to cometary dust, despite their easy implementation.

4. 3D MODELS

Finson and Probstein (1968) showed that the unrealistic 2D synchrone–syndyne models can be converted into realistic 3D models when we associate a synchronic tube to each synchrone, whose width is given by the dust ejection velocity at the synchrone time, or a syndynamic tube to each syndyne, whose width is given by the dust ejection velocity of the syndyne $\beta$ value. In the synchronic approach, they computed in an analytical way the dimensionless sky surface density of a synchronic tube:

$$dD(x, y, t) = \frac{F(t, 1 - \mu) dt}{2 (t_0 - t) v(t, 1 - \mu) \frac{ds}{d(1 - \mu)}}$$

so that the brightness of the whole tail is simply proportional to the numerical time integral of Eq.(12).

Eq.(12) is derived from an analytical integration which is correct if the following condition (named by Finson and Probstein, although quite improperly, hypersonic) is satisfied

$$\frac{ds}{d(1 - \mu)} \gg \frac{(t_0 - t) v(t, 1 - \mu)}{1 - \mu}$$

This usually happens in the outermost dust tail only. In Eqs.(12) and (13), $s$ is the parametric coordinate along the synchrone and $v(t, 1 - \mu)$ the dust ejection velocity. In Eq.(12), the synchronic (or syndynamic) tubes are assumed to have circular sections, because the dust tail is supposed to be built–up by dust shells which keep their spherical expanding shape over all the time, so that significant tidal effects due to the solar gravity are neglected. The spherical shell assumption implies that the dust ejection is isotropic, another strong approximation. When the numerical integration of Eq.(12) is performed, the fit of the tail brightness data is performed by trials, so that we cannot ensure the uniqueness of the obtained dust loss rate, ejection velocity and size distribution. All these approximations make the Finson–Probstein model a first order model unable to give realistic estimates of the dust parameters. Fulle (1987, 1989) developed an inverse Monte–Carlo dust tail model taking into account all the improvements introduced by Kimura and Liu (1977) and avoiding all the limitations and approximations of the Finson–Probstein model:

(i) It computes the rigorous heliocentric Keplerian orbits of millions of sampling dust grains, so that the spherical shell approximation is avoided.

(ii) It performs both the size and time integral by means of numerical methods, so that the condition imposed by Eq.(13) is avoided. In this way, it can fit not only the external tail, but also the inner one, as close as possible to the dust coma, where the largest grains usually remain.

(iii) It takes into account anisotropic dust ejections.
(iv) It is consistent with wide dust velocity distributions (Fulle 1992) simulating 3D dust drag.

(v) It avoids the trial and error procedure typical of the original Finson–Probstein model, by means of inverse ill posed problem theory. In this way, the uniqueness of the results (impossible to be established in the original Finson–Probstein approach) is recovered in the least square fit sense.

Within the inverse Monte–Carlo dust tail model, the dust ejection velocity, loss rate and size distribution are obtained by means of the minimization of the following functional

\[(KF - D)^2 + (RF)^2 = \text{min}\]  

(14)

where \(K\) is the kernel matrix provided by the dust tail model and defined in Eq.(2), \(F\) [solution vector defined in Eq.(8)] the output of the inversion of Eq.(14), \(D\) the dust tail dimensionless sky surface density [data vector defined in Eq.(2)], and \(R\) a regularizing constraint to drop noise and negative values in the solution \(F\). The inverse Monte–Carlo approach was applied to tens of dust tails: the results regarding the DSD (Table 1) and the dust mass loss rate will be discussed in Secs.5 and 6. In general, dust ejection velocities show time and size dependences more complex than predicted by 1D coma models of dust–gas interaction. In particular, the velocity of large grains with respect to small ones seems higher than expected. This result was confirmed by independent tail models which left free such a parameter (Waniak 1992, 1994). This points out that dust–gas interaction must be treated by 3D coma models to predict reliable dust ejection velocities. The outputs of inverse tail models always resulted insensitive to the assumed dust ejection anisotropy. This fact confirms that tail shapes have lost memory of the details of the unknown boundary conditions of the 3D gas drag in the coma.

Other Monte–Carlo dust tail models were developed following other approaches. Eq.(2) allows us to fit more details of the input tail data by means of a time and size dependent dust cross section [i.e., by \(f(t, 1 - \mu)\)]. If the DSD is assumed constant in time to stabilize the model outputs, then it may become impossible to perfectly fit the tail data. The assumption of a DSD constant in time is commonly followed: Lisse et al. (1998) analysed dust tails at mm-wavelengths (data by COBE satellite) with a poor angular resolution (20 arcmin), so that rough assumptions on DSD \(f(1 - \mu) = (1 - \mu)^{-1}\) implying \(\alpha = -3\) allowed them to fit the input images. Waniak (1994), adopting a constant \(\alpha \approx -3\), improved the tail fit of Comet Wilson 1987VII by means of a detailed dust ejection pattern. However, the fact that this pattern ejects dust mainly on the nucleus night side may point out that time changes of the DSD are more probable sources of dust tails shapes.

5. THE DUST SIZE DISTRIBUTION (DSD) IN COMETS

DSD plays a crucial role when we describe dust in comets. For instance, it is widely believed that in dust comae we see mainly grains of sizes close to the observation wavelength. It is easy to conclude [see Eq.(15) in Sec.6.1] that the coma brightness depends on the largest ejected size if \(\alpha > u - 3\), where \(u\) is the power
index versus \(\rho_d d\) of the dust velocity. The assumptions \((\alpha = -3.0 \text{ and } u = -0.5)\) by Lisse et al. (1998) imply that, if \(\alpha > -3.5\), then the dust coma brightness is dominated by the largest ejected boulders in optical wavelengths too. Since dynamical models in comae of micron–sized grains or of meter–sized boulders are quite different, it is crucial to understand which dust we are observing. The dust mass depends on the largest or the smallest ejected grains according to the actual DSD: it is fundamental to define the size range to which the published results are related, although this is commonly not done. For usual \(\alpha > -4\), the dust mass diverges if we allow it to reach the nucleus size. It is impossible to compare dust masses without knowing to which largest size they refer.

The inverse tail model was applied to tens of cometary dust tails, providing results concerning dust sizes between 1 \(\mu\)m and about 1 cm (Table 1). The DSD often showed large changes in times: since most output noise affects the DSD time–dependence, in Sec.6.2 we will pay special care to test this particular output. The time–averaged DSD is much more stable: in almost all comets it was characterized by an index \(\alpha \approx -3.5\). This index is typical of a population of bodies with its evolution dominated by collisions: were this index further confirmed, it would suggest that we do not observe in comets the pristine dust population of the presolar nebula. The \(\alpha = -3.5\) value is the most critical for models of coma features, because it implies that the coma brightness depends uniformly on all dust sizes when we accept the \(u = -0.5\) value provided by 1D models of dust–gas interaction (Crifo 1991) at \(d >> 1\mu\)m. In other words, only coma models taking into account also meter–sized boulders can provide reliable fits of the observed coma features. Luckily, inverse tail models suggest \(-0.5 < u < 0\), in which case the dust coma brightness depends again on the micron–sized grains. However, all coma models adopt \(u = -0.5\). While it is possible to define a theory of dust tails able to disentangle the dependence of the necessarily many dust parameters on the available data, this becomes impossible in models of inner dust comae.

A cornerstone defining DSD in comets is given by the only available in–situ data we have so far: the results of the DID experiment on Giotto (McDonnell et al. 1991) during the 1P/Halley’s flyby. The power index fitting the DSD at the nucleus, traced back from the DID fluence in terms of purely radial expansion, is \(\alpha \approx -3.7\) for dust masses between \(10^{-14}\) and \(10^{-3}\) kg. This slope is also consistent with the impact of the 1 g grain which damaged the probe itself. However, this is exactly an example of how models are crucial to interpret dust data. The fit of the same data by means of a rigorous dust dynamical model completely changed the results (Fulle et al. 2000). The DID fluence is consistent with any \(-2.5 < \alpha < -3.0\), strongly supporting the possibility taken into account by Lisse et al. (1998): in the coma we may commonly observe meter–sized boulders only. The same model inferred also the dust to gas ratio (\(3 < \text{DGR} < 40\) for dust masses up to 1 g) and the dust geometric albedo (\(0.01 < A_p < 0.15\)) by means of the Optical Probing Experiment data, with a correlation between \(A_p\) and \(\rho_d\): \(\rho_d = 2500 A_p\) kg m\(^{-3}\). Tail models and DID experiment agree on these conclusions: (i) \(\alpha >> -4\); (ii) the dust mass
(we cannot exclude the light flux too) depends on the largest ejected boulders; (iii) the DGR is larger than one for sizes larger than 1 cm.

All available results confirm that the dust mass ejected by comets is dominated by the largest ejected boulders. Radar observations (Harmon et al. 1989) operating at cm wavelengths and coma observations at mm wavelengths provide first quality constraints to this conclusion, because they directly observe grains of a size close to the observation wavelength if $\alpha < -3.5$, while they observe dust larger than the observation wavelength if $\alpha > -3.5$. Observations at mm–wavelengths of P/Swift–Tuttle (Jewitt 1996) and C/Hyakutake 1996B2 (Jewitt and Matthews 1997) provided mass loss rates 7 and 10 times higher, respectively, than tail models (Fulle et al. 1994, 1997). Therefore, observations at mm–wavelengths suggest than $\alpha$ was higher than $\alpha = -3.3$ and $\alpha = -3.6$, respectively, provided by the tail model for the two comets (Table 1). We must conclude that the dust coma brightness of these two comets was dominated at all observation wavelengths by the largest ejected boulders if $\alpha = -0.5$.

Is a power law a proper function to describe the DSD? We remind that this assumption is usually adopted by direct tail models only (e.g. Lisse et al. 1998). Inverse tail model do not assume that DSD is a power law: they leave it completely free, sampling the DSD in $\beta$–bins. Then, the output describing the DSD is fitted by a power law, to offer a DSD easily understandable. Sometimes, the direct $f(t, 1 - \mu)$ output was provided (e.g. Fulle et al. 1998). Anyway, we have no DSD data so precise to require more than a power law in order to fit them: a power law DSD is consistent with the DID fluence of 1P/Halley, which is not a power law of the dust size (Fulle et al. 2000). Other functions were suggested to describe the DSD of cometary dust. Hanner (1984) proposed a more complex function, which becomes a simple power law at the largest sizes ($d > 1$ mm), and drops rapidly to zero at submicron grains. This function was used to fit the IR photometry and spectra of comets. These observations are unable to detect submicron grains, so that this is a typical example of “absence of evidence” interpreted as “evidence of absence”: in fact, Giotto showed that in 1P/Halley submicron grains were more abundant than larger ones (McDonnell et al. 1991). This is probably true for all comets: many authors pointed out that every DSD discussed in this review is consistent with all available IR data (Crifo 1987, Greenberg and Li 1999). Dust tail models and in–situ data provide much better constraints to the DSD than IR spectra, and show that a power law defined on a precise size interval is the best approach to describe the DSD in comets, in order to avoid misleading conclusions suggested by more complex size functions.

It would seem obvious that a comet is defined “dusty” according to its dust to gas ratio. In other words, we would like that the higher the index $\alpha$, the higher the released dust mass, the more “dusty” the comet is defined. This is not the case. Usually, a comet is defined “dusty” according to features of its IR spectrum or its polarization, facts easy to be observed, but hard to be related to the actual released dust mass. In particular, when the silicate feature at 10 $\mu$m is strong (Lisse 2002), or when
the highest polarization is higher than 20% (Levasseur–Regourd et al. 1996), then the comet is defined “dusty”. Both these features refer to the actual population of micron–sized grains, which we have seen to be unrelated to the total released mass. If the DSD has a turning point at some size larger than 10 \( \mu \text{m} \), where at smaller sizes \( \alpha \) becomes larger than the 1P/Halley’s index, then the relative production of micron–sized grains drops compared to 1P/Halley. In this case, the comet is defined less “dusty” than 1P/Halley, although its \( \alpha \) from mm to meters may be much higher, with a much higher released dust mass.

6. CONSTRAINTS TO THE OUTPUTS OF 3D TAIL MODELS

6.1 The dust coma equivalent size \( Af\rho \)

The model outputs \( F \) and \( v \) can be compared to the observed dust coma brightness. This is usually measured by means of the \( Af\rho \) quantity (A’Hearn et al. 1984), which is the size of the equivalent dust disk scattering the same light of the observed coma:

\[
Af\rho(t) = 2\pi(\gamma r)^2 \frac{10^{0.4[m_0-m(x,y)]}}{D(x,y)} \int_0^\infty \frac{F(t,1-\mu)}{v(t,1-\mu)} d(1-\mu) \tag{15}
\]

where all the quantities are defined in Sec.2. We point out that Eq.(15) is completely parameter–free: given the outputs \( F \) and \( v \) of the tail model, there is no way to adjust the tail model to fit the observed \( Af\rho \) values. However, the integral of Eq.(15) can be computed on a finite \( \beta \) range, while \( Af\rho \) is measured observing all the ejected dust sizes in the dust coma. Therefore, if the size range adopted to compute Eq.(15) loses dust sizes reflecting a significant light fraction, then tail models can provide only a lower limit of the actually observed \( Af\rho \). In any case, \( Af\rho \) computed by means of Eq.(15) was always consistent with that observed (Fulle et al. 1998, Fulle 2000).

Eqs.(9) and (15) point out that \( Af\rho \) has little to do with the dust mass loss rate. Nevertheless, it is commonly named dust loss rate, despite its dimensions, and is commonly related to the water loss rate to get odd (at least from a physical point of view) dimensional ratios. It is obvious that comets with an higher \( Af\rho \) can eject less dust mass: this depends on the DSD and dust velocity. Moreover, the time evolution of \( Af\rho \) can be unrelated to the loss rate time evolution, depending on time changes of the DSD and velocity. \( Af\rho \) is a very high quality constraint to physical models of comae and tails, but nothing more.

6.2 DSD time variability

So far, only inverse dust tail models take into account the possibility that dust is ejected from the cometary nucleus with a DSD which may change in time. This is surprising independently of the high or low probability that this really happens in comets, but simply because most papers on cometary dust invoke dust fragmentation to explain observations. It is obvious that fragmentation implies a time
evolution of the DSD, and that a model taking into account a time–dependent DSD is more general than another taking into account dust fragmentation only. So far, no consistent models of dust fragmentation were developed (Crifo 1995). Combi (1994) developed a direct dust tail model with fragmentation, which was based on consistent fits of both the dust coma and tail. Many tests performed by means of the inverse tail model adopting many $u$ values showed that such a consistent fit of both the dust coma and tail simply requires $-0.5 < u < 0$. Fulle et al. (1993) showed that dust fragmentation is a possible (not unique) explanation of $-0.5 < u < 0$.

While inverse dust tail models can provide a stable time–averaged DSD, the time–dependent DSD is affected by noise. It is not easy to establish if large and systematic changes of the DSD are real or simply due to output noise. The most elegant solution is to find independent observations suggesting systematic changes of the ejected dust population in agreement with the time–evolution of the DSD provided by inverse tail models. These models applied to C/Hyakutake 1996B2 provided an $\alpha$ value dropping suddenly from a roughly constant value $\alpha = -3$ to $\alpha = -4$ at mid April 1996 (Fulle et al. 1997), in perfect agreement with the time evolution of IR spectra: no silicate feature was detected before mid April, when a strong 10 $\mu$m line appeared (Mason et al. 1998). This IR spectral evolution was interpreted in terms of dust fragmentation exposing small silicatic cores to the Sun radiation, embedded in larger carbonatic matrices before mid April. The inverse tail model applied both to optical data (Fulle 1990) and to IR thermal data (Epifani et al. 2001) regarding two different perihelion passages of Comet 2P/Encke provided a similar drop from $\alpha = -3$ to $\alpha = -4$ during the first three weeks after perihelion. Already this coincidence forces us to exclude that this DSD time–evolution is due to output noise. Moreover, these three weeks exactly match the seasonal night of the most active nucleus hemisphere suggested by Sekanina (1988) to explain the comet photometry and coma shape evolution of 2P/Encke.

7. FINE STRUCTURES IN TAILS

7.1 Neck–Lines

In several cases, dust tails maintain memory of the dust ejection over more than half of the comet orbit: during so long times, the tidal effects of solar gravity become significant. Kimura and Liu (1977) pointed out that the dust motion is heliocentric, so that at the ejection a dust grain can be considered at the first node of its heliocentric orbit. Every heliocentric orbit has its second node 180° away from the first, where the dust grain orbit must necessarily cross again the comet orbit. When we consider a Finson–Probstein dust shell, all these grains, ejected at the same time, will have their second orbital node approximately at the same time, i.e. after 180° of orbital anomaly, where the spherical dust shell will shrink into a 2D ellipse flat on the comet orbital plane: a shape far indeed from a sphere. When we consider a shrunk synchronous tube, we obtain a 2D structure, named Neck–Line. When the Earth crosses the comet
orbit, the Neck–Line appears in the sky as a straight line much brighter than the surrounding dust tail, because all the synchronic tube is shrunk in a infinitesimal sky area (Fig. 3). By means of the Neck–Line model, Kimura and Liu perfectly fitted the perspective antitail of Comet Arend Roland 1957III, which appeared as a bright spike many millions km long and pointing towards the Sun, thus avoiding unrealistic explanations based on dust ejected at zero velocity from the parent coma.

Neck–Lines were observed in the Great Comet 1910 I, in Comets Arend–Roland 1957III, Bennett 1970II, 1P/Halley 1986III, Austin 1990V, Levy 1990XX and Hale–Bopp 1995O1. It must be pointed out that Neck–Lines can appear only after perihelion, a fact rigorously verified by all the registered apparitions. Due to the particular perspective conditions, in Comets Arend–Roland and Levy the Neck–Line appeared as a perspective anti–tail. In all other comets, it appeared superimposed to the main dust tail as a bright and straight linear feature. Since a Neck–Line is built–up by the collapse of the dust shells onto 2D ellipses, the ellipses composed of the largest grains, approximately centered on the comet nucleus, are placed half out of the comet orbit and half inside it. Due to the long travel time of the dust grains (usually months), these flat ellipses may reach huge dimensions, up to $10^6$ km: when the Earth crossed the orbit plane of Comets Bennett, 1P/Halley, Austin and Hale–Bopp, the half ellipse inside the comet orbit was seen as a bright spike pointing to the Sun, which was in fact a real (non perspective) anti–tail. Since these real anti–tails are composed of the largest grains ever observed in dust tails, Neck–Line observations provide unique information on the ejection velocity and size distribution of grains larger than cms.

Fulle and Sedmak (1988) have obtained analytical models of Neck–Lines, so that the Neck–Line photometry provides the $\beta$ distribution and the dust ejection velocity at the ejection time $t$ of the Neck–Line. The Keplerian dynamics of the grains in space allow us to compute the geometric Neck–Line parameters $a$ and $b$ (related to the major and minor axes, respectively, of the ellipses composing the Neck–Line) and $s$ (the parametric coordinate along the Neck–Line axis $x$). If the velocity condition

$$v(1 - \mu) << (1 - \mu) s a$$

is satisfied, then the dimensionless sky surface density in the Neck–Line is

$$D(x, y) = \frac{F(t, 1 - \mu)}{s v(1 - \mu)} \left[ \frac{a x}{v(1 - \mu)} \right] \exp \left( -\frac{b^2 y^2}{v^2(1 - \mu)} \right)$$

From Eq.(17), it is apparent that the Neck–Line width along the $y$ axis provides a direct measurement of the $\beta$ dependence of the dust velocity at cm sizes. These are the only available direct observations of such a dependence, which is in agreement with the results of 3D inverse dust tail models: $-0.5 << \mu < 0$. The obtained $\beta$ distributions confirm that most of the dust mass is released in the form of the largest ejected grains.
7.2 Striae

In the brightest comets (e.g. Great Comet 1910I, Comets Mrkos 1957V, West 1976VI, Hale–Bopp 1995O1), the usually structureless dust tail shows detailed substructures of two kinds, namely synchronic bands and striae. The synchronic bands are streamers pointing to the comet nucleus, with the axis well fitted by synchrones. There is general agreement that they are due to time changes of the dust loss rate or of the DSD, so that the dust cross section is larger in the synchronic bands than outside. On the contrary, the striae do not point to the comet nucleus (Fig. 4), and they are neither fitted by synchrones nor by syndynes. There is general agreement that striae are due to instantaneous fragmentation of larger parents, so that the striae are synchrones originating not from the comet nucleus, but from the tail point where the parent was placed at the time of fragmentation. This explains well the orientation of the striae, which always point between the comet nucleus and the Sun.

Available models of bands and striae were performed by adopting 2D synchrone–syndyne models, so that the available quantitative results may be affected by significant errors: the dust ejection velocity from the coma is assumed to be zero, in contrast with all available information on the dust dynamics. For striae models in particular, which are very sensitive to the $\beta$ value of the parent grain, this assumption might significantly affect the results: the $\beta$ value of the parent is easily obtained by the stria origin in a 2D model, while many different $\beta$ values are consistent with the stria origin in a 3D model. Sekanina and Farrell (1980) obtained that the submicron–sized fragments observed in the striae of Comet West 1976VI had a $\beta$ very similar to their much larger parents. This result would imply that the parents must be very elongated chains of submicron–sized grains. However, this implies that also the drag by gas on these chains was correspondingly high (both gas drag and radiation pressure depend on the parent cross section): these chains must have been ejected from the coma exactly at the gas velocity, so that they would have been diluted over huge $10^6$ km–sized shells. The stria origin cannot constrain either the fragmentation time or the $\beta$ value of the parent grain.

7.3 Sodium tail

Although spectroscopic observations suggested the presence of tail extensions of the well known neutral sodium coma of Comets 1910I and Arend–Roland 1957III, the first clear images of a huge sodium tail $10^7$ km long, well separated in the sky from classical dust and plasma tails, were obtained during the 1997 passage of Comet Hale–Bopp 1995O1 (Cremonese et al. 1997). The images were taken by means of interference filters centered on the sodium D lines at 589 nm, and the tail did not appear in simultaneous images taken on the H$_2$O$^+$ line, showing a well developed ion tail. The sodium tail appeared as a straight linear feature placed between the ion tail and the prolonged radius vector. Simultaneous spectroscopic observations allowed to measure the radial velocity of the neutral sodium atoms along the tail. It resulted
that a syndyne of $\beta = 82$ best fitted both the sodium tail axis orientation and the radial velocities along the tail. This was the first time that the $\beta$ parameter was best constrained by means of radial velocity measurements. The sodium neutral atoms lightens thanks to fluorescence: absorbed UV solar photons spend their energy to put the external sodium electrons in the most external orbitals. Since the solar photons are absorbed by the sodium atoms, their momentum is necessarily transferred to the sodium atoms, so that

$$\beta = \frac{h g (1\text{AU})^2}{\lambda GM_{\odot} m}$$ \hspace{1cm} (18)

where $h$ is the Planck constant, $g$ the number of solar photons captured in the unit time by a sodium atom at 1 AU (or photon scattering efficiency in the sodium D lines), $\lambda$ the wavelength of the sodium D lines, and $m$ the tail particle mass. In perfect agreement with the theoretical computations, the observed $\beta = 82$ provides $g = 15 \text{ s}^{-1}$ when we assume the atomic sodium mass for $m$. Therefore, the sodium tail is composed of sodium atoms and not of sodium molecules. The sodium atoms in space have a short lifetime, mainly due to photoionization. The sodium tail brightness along its axis $x$ provides an unique measurement of the sodium lifetime $\tau$. Since the sodium tail axis can be best approximated by a syndyne, the whole sodium tail can be modelled by means of a syndynamic tube, whose photometric equation was computed by Finson and Probst (1968). When we consider the sodium lifetime against photoionization, the sky surface density of sodium atoms is

$$\delta(x, y) = \frac{\dot{N}}{2 v \left[ t_o - t(x) \right]} \exp\left( \frac{-t_o - t(x)}{\tau} \right) w(x)$$ \hspace{1cm} (19)

where $\dot{N}$ is the sodium loss rate, $v$ the sodium ejection velocity (related to the sodium tail width), $w$ the sodium radial velocity projected on the sky, and $t$ the time of sodium ejection ($w$ and $t$ are provided by syndyne computations). Eq.(19) perfectly fits the observed brightness on the tail axis $x$ when we assume $\tau = 1.7 \times 10^5 \text{s}$ at 1 AU. This lifetime is three times larger than the value assumed in comet and planetary atmospheric models, as it was already suggested by laboratory measurements by Huebner et al. (1992).

8. REFERENCES


phenomenon. *Astron. J.,* 85, 1538–1554

100, 154–161


<table>
<thead>
<tr>
<th>TABLE 1. Dust Size Distributions in Comets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comet</td>
</tr>
<tr>
<td>2P/Encke</td>
</tr>
<tr>
<td>6P/D’Arrest</td>
</tr>
<tr>
<td>10P/Tempel2</td>
</tr>
<tr>
<td>26P/Grigg–Skjellerup</td>
</tr>
<tr>
<td>29P/SW1</td>
</tr>
<tr>
<td>46P/Wirtanen</td>
</tr>
<tr>
<td>65P/Gunn</td>
</tr>
<tr>
<td>P/Swift–Tuttle</td>
</tr>
<tr>
<td>2060 Chiron</td>
</tr>
<tr>
<td>Seki–Lines 1962III</td>
</tr>
<tr>
<td>Kohoutek 1973XXII</td>
</tr>
<tr>
<td>Wilson 1987VII</td>
</tr>
<tr>
<td>Bradfield 1987XXIX</td>
</tr>
<tr>
<td>Liller 1988V</td>
</tr>
<tr>
<td>Austin 1990V</td>
</tr>
<tr>
<td>Levy 1990XX</td>
</tr>
<tr>
<td>Hyakutake 1996B2</td>
</tr>
</tbody>
</table>

Time dependent [$\alpha(t)$] and time–averaged ($\alpha_m$) DSD power index evaluated for various comets within a
$\beta$ range (defined by $\beta_{\text{min}}$ and $\beta_{\text{max}}$) by means of the inverse tail model (adapted from Fulle 1999).

9. FIGURES CAPTIONS

Fig. 1 – A typical structureless dust tail: Comet Hale–Bopp 1995O1 on 1.84 UT May 1997. The blue tail
pointing to the left is the ion tail, while the whitish tail pointing to the top is the dust tail. Copyright
and observer Marco Fulle.

Fig. 2 – Synchrone–syndyne network for Comet Hale–Bopp 1995O1 on 3 January 1998, when a neck–line
was observed during the Earth crossing of the comet orbital plane (Fig. 3). The sun was located exactly
towards the $-y$ direction and the phase angle of the comet at the observation was $\phi = 14.4^\circ$. The image
plane corresponds to the comet orbital plane; the Earth direction is towards the bottom right forming an
angle $\phi$ with the $-y$ axis. The dotted line is the comet elliptical orbit, which, together with the $y$ axis,
divides the plane in the four sectors discussed in Sec. 1. The continuous lines are syndynes characterized by the parameter \( \beta = 1 - \mu = 1, 0.3, 0.1, 0.03, 0.01, 0.003, 0.001 \) and 0.0003, rotating clockwise from that closest to the \( +y \) direction, respectively. The dashed lines are synchrones characterized by the ejection times 250, 200, 100, 0, –100, –200 and –300 days with respect to perihelion, rotating clockwise from the closest one to the \( +y \) direction, respectively. The three–dotted and dashed line is the neck–line axis, approximately corresponding to the synchrone ejected 43.8 days before perihelion. Because of the Earth position, it is evident that the neck–line was observed as a spike pointing towards the anti–solar direction (Fig. 3); its prolongation in the \( -x \) direction was observed as a bright spike–like real anti–tail (Fig. 3).

Fig. 3 – Neck–line observed in Comet Hale–Bopp 1995O1 on 5 January 1998 with the ESO 1m Schmidt Telescope. The original image was filtered (unsharp masking) to enhance the spike features: the real anti–tail pointing towards the sun (bottom) and the neck–line pointing in the opposite direction. Image Copyright European Southern Observatory, observer Guido Pizarro.

Fig. 4 – Striae observed in the dust tail of Comet Hale–Bopp 1995O1 on 16.15 UT March 1997. The original image was filtered (unsharp masking) to enhance the striae in the whitish dust tail, which do not point towards the comet head. The blue tail on the left is the ion tail. Copyright and observer Marco Fulle.

### 10. LIST OF SYMBOLS

- \( \alpha \) Power index of the DSD versus \( (\rho_d d) \) (dimensionless).
- \( \beta = 1 - \mu \) Ratio between solar radiation pressure and gravity forces (dimensionless).
- \( \gamma \) A radiant (arcsec).
- \( \delta(x,y) \) Sodium surface density (m\(^{-2}\)).
- \( \phi \) Phase angle (deg).
- \( \lambda \) Observation wavelength (m)
- \( \mu = 1 - \beta \) Correction factor of the solar gravity force to which a grain is subjected (dimensionless).
- \( \rho_d \) Dust bulk density (kg m\(^{-3}\)).
- \( \sigma(t,1-\mu) \) Cross section of a mean dust grain (m\(^2\)).
- \( \tau \) Sodium lifetime against photoionization (s).
- \( a, b \) Neck–Line ellipse parameters (s\(^{-1}\)).
- \( A_f \rho \) Measure of the dust coma brightness (m).
- \( A_p(\phi) \) Dust geometric Albedo times the phase function (dimensionless).
- \( A_U \) Astronomical Unit (m).
- \( B \) Unit cross section grain brightness (W m\(^{-2}\) Hz\(^{-1}\) sr\(^{-1}\)).
- \( B_\nu(T) \) Planck function (W m\(^{-2}\) Hz\(^{-1}\) sr\(^{-1}\)).
c Light speed (m s\(^{-1}\)).

\(C_{pr}\) Proportionality constant in the relation between \(\beta, \rho_d, d\) and \(Q_{pr}\) (kg \(m^{-2}\)).

d Mean size of a compact dust grain (m).

\(D(x, y)\) Dust surface density weighted by dust cross section (dimensionless).

DGR Dust to Gas Ratio (dimensionless).

DSD Differential Dust Size Distribution (m\(^{-1}\)).

\(E_\odot\) Mean solar radiation (W).

\(f(t, 1 - \mu)\) \(\beta\) distribution (dimensionless).

\(F(t, 1 - \mu)\) Output of tail models (m\(^2\) s\(^{-1}\)).

\(g\) Photon scattering efficiency (s\(^{-1}\)).

\(g(t, \rho_d d)\) DSD versus \((\rho_d d)\) (m\(^2\) kg\(^{-1}\)).

\(G\) Gravitation constant (m\(^3\) kg\(^{-1}\) s\(^{-2}\)).

\(h\) Planck constant (kg m\(^{-2}\) s\(^{-1}\)).

\(I(x, y)\) Tail surface brightness (W m\(^{-2}\) Hz\(^{-1}\) sr\(^{-1}\)).

\(k\) Normalization factor of the size distribution \(g\) (dimensionless).

\(K(x, y, t, 1 - \mu)\) Dust surface density from tail models (m\(^{-2}\)).

\(m\) Sodium atomic mass (kg).

\(m(x, y)\) Tail surface magnitude (mag arcsec\(^{-2}\)).

\(m_\odot\) Sun magnitude (mag).

\(M_\odot\) Solar mass (kg).

\(\dot{M}(t)\) Dust mass loss rate (kg s\(^{-1}\)).

\(\dot{N}(t)\) Dust number loss rate (s\(^{-1}\)).

\(Q_{pr}\) Scattering efficiency for radiation pressure (dimensionless).

\(r\) Sun–Comet distance in AU (dimensionless).

\(R\) Regularizing constraint (m\(^{-2}\) s).

\(s\) Parametric sky coordinate (m).

\(S_\nu(x, y, T)\) Tail thermal flux (W m\(^{-2}\) Hz\(^{-1}\) sr\(^{-1}\)).

\(t\) Time (s).

\(t_o\) Observation time (s).

\(T\) Grain temperature (°K).

\(u\) Power index of the dust ejection velocity versus \((\rho_d d)\) (dimensionless).

\(v(t, 1 - \mu)\) Dust ejection velocity from the coma (m s\(^{-1}\)).

\(w\) Sodium radial velocity (m s\(^{-1}\)).

\(x, y\) Sky coordinates (m).
prolonged radius vector ($10^6$ km)