3. Classical and Quantum Worlds

1. Kepler’s & Galileo’s Laws
2. The Newtonian Era
3. Down the Rabbit Hole
Newton consolidated a wide range of observations and empirical rules into a few simple and straightforward laws of motion. For several centuries thereafter, astronomers and physicists were kept busy working out the consequences of these laws. But roughly one hundred years ago, physicists discovered the world was not as straightforward as they'd believed.
I. KEPLER’S AND GALILEO’S LAWS

a. Using Kepler’s laws

b. Galileo’s laws of motion

c. Orbital motion
Using Kepler’s Laws

**Law I:** Planets move in elliptical orbits with the Sun at one focus.

Closest to Sun at **perihelion**, furthest from Sun at **aphelion**.

Width of ellipse is $2a$, where $a$ is the **semi-major axis**.

**Law II:** A line from the Sun to a planet sweeps out equal areas in equal times.

In other words, $(\text{area})/(\text{time})$ is a constant, now known as the **angular momentum**.

**Law III:** The quantity $P^2 / a^3$, where $P$ is orbital period and $a$ is semi-major axis, is the **same** for all planets.

If $P$ is measured in years and $a$ is measured in AU, then $P^2 = a^3$. 
### Law III

<table>
<thead>
<tr>
<th>Planet</th>
<th>$a$ (AU)</th>
<th>$P$ (yr)</th>
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<tbody>
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Difference is less than expected error.
Uranus orbits the Sun at an average distance of \( a = 19.23 \) AU. What is its orbital period \( P \)?

\[
P = \sqrt{a^3} = \sqrt{(19.23 \times 19.23 \times 19.23)} = \sqrt{71111} = 84.3
\]

Uranus takes 84.3 yr to orbit the Sun.

Neptune takes 164.79 yr to orbit the Sun. What is its average distance \( a \)?

\[
a = \sqrt[3]{P^2} = \sqrt[3]{(164.79 \times 164.79)} = \sqrt[3]{27156} = 30.1
\]

Neptune’s average distance from the Sun is 30.1 AU.
1. An asteroid orbits the Sun at a distance of $a = 4$ AU. What is its period $P$?

A. $P = 2$ yr
B. $P = 4$ yr
C. $P = 6$ yr
D. $P = 8$ yr
E. $P = 10$ yr

Hint: $P = \sqrt{a^3} = \sqrt{4^3} = \sqrt{64}$
Using Law II: Why the Moon “Wobbles”

The Moon’s orbital and rotation periods are equal (~27.3 days).

If it’s orbit was a perfect circle, the Moon would not wobble; the same side would always face us precisely.
Using Law II: Why the Moon “Wobbles”

The Moon’s orbital and rotation periods are equal (~27.3 days).

The dashed lines divide the area inside the Moon’s orbit in four equal parts and thus four equal times (Law II).

In each of these times the Moon rotates by exactly 90°. But to keep exactly the same side toward us, it would have to rotate faster when close, and slower when far.
2. The dashed lines now divide the area inside the Moon’s orbit into five equal parts. How long does it take the Moon to sweep out one of these “slices”?

A. One-half an orbital period.
B. One-third an orbital period.
C. One-quarter an orbital period.
D. One-fifth an orbital period.
E. One-sixth an orbital period.
3. Given that the Moon’s orbital and rotation periods are equal, how much does the Moon rotate in the time it takes to sweep out one slice?

A. One-half a full circle.
B. One-third a full circle.
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How to Get Home from Mars

Assume planets have circular orbits with $a_m = 1.52$ AU and $a_e = 1$ AU.

Homeward orbit is ellipse (Law I) with aphelion at 1.52 AU and perihelion at 1 AU.

Semi-major axis $a = (a_m + a_e) / 2 = 1.26$ AU.

Orbital period (Law III) is $P = \sqrt{a^3} = \sqrt{2.00} = 1.41$ yr.

Time to get home is half the orbital period (Law II), or 0.71 yr.
Together, Kepler's laws completely describe the motion of a planet.

- Law I gives the shape of an orbit.
- Law II gives the speed at each point along an orbit.
- Law III connects sizes and periods of orbits.

These laws also describe orbits of comets, asteroids, and interplanetary spacecraft, moons and satellites of planets, and even the orbits of double stars!
Galileo’s Laws of Motion: Inertia

A moving body moves in a straight line with a constant speed unless acted on by an outside force.
Galileo’s Laws: Falling Bodies I

All bodies, regardless of weight, fall at the same speed.
4. What would happen if you tried this experiment on Earth?

A. The hammer would fall faster.
B. The feather would fall faster.
C. Both would fall at the same speed.
D. The feather would blow away.
E. The hammer would hit your toe.
Galileo’s Laws: Falling Bodies II

The distance $D$ a body falls is proportional to the square of the time $t$ it has been falling:

$$D = \frac{1}{2} g t^2$$

Here $g = 9.8 \text{ m/s}^2$ is the acceleration due to gravity at the Earth’s surface.
Galileo’s Laws: Relative Motion

Galileo argued that motion is relative: someone moving at constant speed and direction can only detect their motion by looking at the outside world. For example, on a plane which is flying smoothly you have no way of knowing that you're in motion without looking out the window.
A ball released 200 km above the Earth falls

18 km after 60 s
72 km after 120 s
162 km after 180 s

and hits the surface after 200 s.
Orbital Motion: Low Earth Orbit

If the ball is released with some sideways velocity, it continues moving sideways as it falls (law of inertia).
Orbital Motion: Low Earth Orbit

If we increase the sideways velocity, the ball lands further and further away.

Galileo's work on projectile motion
Orbital Motion: Low Earth Orbit

If the sideways velocity is about 8 km/s, the Earth curves away *just enough* that the ball remains 200 km above the surface.

The ball is in a *circular orbit*! It takes 88.5 minutes to circle the Earth — typical for a low Earth orbit.
5. What if the sideways velocity is more than 8 km/s?

A. The ball will slow down to 8 km/s.
B. The ball’s orbit will be an ellipse.
C. The ball will spiral away from Earth.
D. The ball will spiral into Earth.
E. The ball will escape Earth forever.
Orbital Motion: Orbits Compared

Do the ball and the Moon both obey Kepler’s Law III?

\[ a_{\text{ball}} = 200 \text{ km} + R_{\oplus} = 6580 \text{ km} \]

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Yes! — difference is less than expected error.
KEPLER’S AND GALILEO’S LAWS: SUMMARY

a. Using Kepler’s laws

Get the **shape** (Law I), **speed** (Law II), and **period** (Law III) of an orbit.

b. Galileo’s laws of motion

(a) Inertia: *keep moving!*, (b) falling bodies: *distance proportional to time squared*, (c) motion is relative.

c. Orbital motion

An orbiting ball falls towards the Earth but never hits — and *the Moon does that too!*
2. THE NEWTONIAN ERA

a. Newton’s laws of motion and gravity
b. Conservation of energy and momentum
Newton’s Laws: Describing Motion

**Speed:**
\[(\text{distance}) / (\text{time})\]

**Velocity:**
speed and direction

**Acceleration:**
change of velocity

Which car is moving at constant *speed*?  \[\text{B}\]

Which car is moving at constant *velocity*? \[\text{none}\]

Which car is *accelerating*? \[\text{all}\]
Newton’s Laws: Acceleration

If an object is under constant acceleration, the change in its velocity is proportional to the time.

E.g., a falling body has a constant acceleration of $g \approx 10 \text{ m/s}^2$ down. If it starts falling from rest at $t = 0$, its downward speed is

$$v = g \ t$$
Newton’s Laws: Mass and Weight

**Mass:**
the amount of “stuff” in an object

**Weight:**
the force needed to keep an object from falling

You can gain or lose weight in an accelerating elevator, but your clothes will be just as tight...
Newton’s Laws of Motion

Law I: An object moves at a constant velocity if the total force on it is zero.

In a plane flying smoothly at constant velocity, the total force on your body is zero.

Law II: To give an object of mass $m$ an acceleration $a$ requires a force $F = ma$.

A pitcher accelerates a baseball with a force dozens of times the weight of the ball.

Law III: For every force there is an equal and opposite reaction force.

By forcing hot exhaust gasses out its engine nozzles, the space shuttle accelerates forward.
Newton’s Law of Gravity

1. *Every* mass attracts *every* other mass.

2. The force $F$ of attraction between a masses $M$ and $m$ separated by distance $d$ is

$$F = G \frac{Mm}{d^2}$$

where $G$ is Newton’s gravitational constant.
Newton’s Law of Gravity

The attraction between ordinary objects is very weak, so a sensitive experiment is needed to detect it.
Newton’s Law of Gravity

The attraction between ordinary objects is very weak, so a sensitive experiment is needed to detect it.
Orbital Motion

Newton showed that bound orbits are always elliptical (or circular), but an object moving fast enough may escape on a *parabolic* or *hyperbolic* orbit.
Orbital Motion: Kepler’s Law III

For *any* two masses $M$ and $m$ orbiting each other, Newton updated Kepler’s Law III:

$$\frac{p^2}{a^3} = \frac{4\pi^2}{G(M + m)}$$

This provides a way of “weighing” planets and stars...
3. DOWN THE RABBIT HOLE

a. The nature of light

b. Atomic structure

c. Spectral lines
The Nature of Light: The Spectrum

White light is a mixture of colors.

The individual colors are pure; they can’t be broken down further.
The Nature of Light: The Spectrum

White light is a mixture of colors.

The individual colors are pure; they can’t be broken down further. Invisible colors exist on either side of the visible spectrum.
The Nature of Light: Waves?

Light behaves like waves: **diffraction**.

Waves passing through a narrow opening spread out.

Experiments showed color is related to wavelength ($\lambda$):

$$\lambda_{\text{blue}} < \lambda_{\text{green}} < \lambda_{\text{red}}$$
The Nature of Light: Waves?

Electromagnetic theory refined the idea of light waves:

These electromagnetic waves were predicted to travel at the speed of light:

\[ c = 3 \times 10^8 \text{ m/s} \]
Visible light is part of the electromagnetic spectrum:

Wavelength ($\lambda$) and frequency ($\nu$) are *inversely* related:

\[ \lambda \nu = c \]
The Nature of Light: Particles?

Light behaves like particles: **photo-electric** effect.

Light knocks electrons free.

Number of electrons depends on *intensity*, energy of electrons depends on *color*:

\[ E_{\text{blue}} > E_{\text{green}} > E_{\text{red}} \]

(If light is waves, *energy* should depend on *intensity.*)
IS LIGHT A

WAVE? wavelength $\lambda$

PARTICLE? energy $E$

EXPERIMENTS SAY

YES! diffraction

YES! photoelectric

??????
The Nature of Light: Photons!

Both descriptions are right!

Photons behave like:
1. **Waves** when traveling through space
2. **Particles** when interacting with matter

The energy carried by a *single* photon is:

\[ E = h\nu = \frac{hc}{\lambda} \]

where \( h \) is **Planck’s constant**.
Atomic Structure: Discovery of the Nucleus

Scattering of “α particles” by thin gold foil:

Most go right through — a few are deflected...

WHY?
An atom’s mass is packed into a tiny, positively charged nucleus, orbited by negatively charged electrons.
Atomic Structure: Electron Orbits

Electrons can only occupy some orbits — all other orbits are forbidden.

In *hydrogen*, the allowed orbits are those which sweep out area per unit time

\[ n \times \frac{h}{m_e} \]

where \( n \) is a *whole number*, \( m_e \) is the electron mass, and \( h \) is Planck’s constant.

Other elements are more complex, but the basic idea still holds.
Atomic Structure: Electron Orbits

Each orbit has a definite energy level.

In hydrogen, orbit \( n \) has energy

\[
E_n = \left(1 - \frac{1}{n^2}\right) \times 13.6 \text{ eV}
\]

where an eV is a unit of energy.

To jump from one level to another requires exactly the right amount of energy.
Spectral Lines: Absorption

Photons arriving with *just* the right energy can make electrons jump to higher level orbits.

For *hydrogen* starting in the $n = 2$ state:
Spectral Lines: Emission

If the electron jumps back down, it radiates a photon of \textit{exactly} the same energy.

For hydrogen starting in the \( n = 3, 4, \) or 5 states:
Types of Spectra

Continuous Spectrum

Emission Line Spectrum

Absorption Line Spectrum
CLASSICAL AND QUANTUM: REVIEW

Kepler’s and Galileo’s Laws

Motion on Earth and in space are related.

The Newtonian Era

Physical laws — including gravity — are universal.

Down the Rabbit Hole

On small scales, nature makes quantum jumps.