Astronomy 241: Review Questions #1  
Distributed: September 26, 2013

Review the questions below, and be prepared to discuss them in class on Oct. 1. Modified versions of some of these questions will be used in the midterm exam on Oct. 3.

1. State Kepler’s three laws, and give an example of how each one is used.

2. You are in a rocket in a circular low Earth orbit (orbital radius \( a_{\text{geo}} = 6600 \text{ km} \)). What velocity change \( \Delta v \) do you need to escape Earth’s gravity with a final velocity \( v_{\infty} = 2.8 \text{ km s}^{-1} \)? Assume your rocket delivers a brief burst of thrust, and ignore the gravity of the Moon or other solar system objects.

3. Consider an airless, rapidly-spinning planet whose axis of rotation is perpendicular to the plane of its orbit around the Sun. How will the local equilibrium surface temperature \( T \) depend on latitude \( \ell \) (measured, as always, in degrees from the planet’s equator)? Neglect heat conduction and internal heat sources.

4. Given that the density of seawater is \( 1025 \text{ kg m}^{-3} \), how deep do you need to dive before the total pressure (ocean plus atmosphere) is \( 2P_0 \), where the typical pressure at the Earth’s surface is \( P_0 = 1.013 \times 10^5 \text{ kg m}^{-1} \text{s}^{-2} \)?

5. An asteroid of mass \( m \ll M_E \) approaches the Earth on a hyperbolic trajectory as shown in Fig. 1. This figure also defines the initial velocity \( v_0 \) and “impact parameter” \( b \). Ignore the gravity of the Moon or other solar system objects.
   (a) What are the energy \( E \) and angular momentum \( L \) of this orbit? (Hint: one is easily evaluated when the asteroid is far away, the other when it makes its closest approach.)
   (b) Find the orbital eccentricity \( e \) in terms of \( M_E, E, L, \) and \( m \) (note that \( e > 1 \)). Then use your results for part (a) to express \( e \) in terms of \( M_E, v_0, \) and \( b \).
   (c) What is the actual pericentric separation \( r_p < b \) of the asteroid’s orbit?
   (d) Long after this encounter, the asteroid will again be moving in a straight line with velocity \( v_0 \). What is the angle \( \theta \) between this line and the asteroid’s initial trajectory?

   ![Figure 1](image)

   Figure 1: An asteroid approaches the Earth. Before being accelerated by the Earth’s gravity, its initial velocity is \( v_0 \); if it continued in a straight line, it would pass the Earth at minimum distance \( b \).
6. The greenhouse skeptic argues “The ‘greenhouse gasses’ in the Earth’s atmosphere are cooler than the surface of the Earth. The Second Law of Thermodynamics says heat can never flow from cold to hot. Thus it’s impossible for greenhouse gasses to warm the Earth.” Your response?

7. The Space Station is in a circular orbit with a radius of \( a_{SS} = 6800 \text{ km} \). You are in a spaceship in the same orbit as the Space Station, but trailing it by a distance of 100 km.
   (a) What happens if you try to catch up by firing your rocket engines in the direction of your orbit?
   (b) How would you actually fire your rockets in order to reach the Space Station?

8. The Moon orbits the Earth with an orbital period \( P_M = 27.3 \text{ day} \) and a semi-major axis \( a_M = 3.84 \times 10^5 \text{ km} \). Using this information, calculate
   (a) the semi-major axis \( a_{gso} \) of a geosynchronous orbit \( (P_{gso} = 1 \text{ day}) \), and
   (b) the orbital period \( P_{leo} \) of a low Earth orbit \( (a_{leo} = 6600 \text{ km}) \).

9. Three of Jupiter’s moons are locked into a resonance: the orbital periods of Io, Europa, and Ganymede have ratios of
   \[
   P_{Io} : P_{Eu} : P_{Ga} = 1 : 2 : 4 .
   \] (1)
   In other words, Io completes four orbits, and Europa completes two, in the time it takes Ganymede to complete one. What is the ratio of their orbital radii \( a_{Io} : a_{Eu} : a_{Ga} \)?

10. We think that Mercury originally had an iron core comprising \( \sim 30\% \) of its mass (much like the Earth). After Mercury differentiated, a giant impact blasted much of the rocky mantle away without seriously disturbing the mass of the core; as a result, Mercury’s core now makes up \( \sim 60\% \) if the planet’s mass.
   (a) Given that Mercury currently has mass \( M_{Me} = 3.3 \times 10^{23} \text{ kg} \), what was its mass before the impact? You may ignore the mass of the impacting body.
   (b) Roughly how much energy would have been required to eject the mantle material Mercury lost?
   (c) Assuming the impacting body had a velocity of 50 km s\(^{-1}\), how massive would it have had to be to deliver the necessary kinetic energy? Compare your result with Mercury’s original mass.

11. Consider a homogeneous planet (i.e., one with constant internal density \( \rho \)) of mass \( M \) and radius \( R \).
   (a) At radius \( r \leq R \), what is the gravitational acceleration \( g(r) \)?
   (b) Use the equation of hydrostatic equilibrium,
   \[
   \frac{dP}{dr} = -\rho g(r) \] (2)
   to calculate the central pressure \( P_c \).
12. Consider a homogeneous planet (i.e., one with constant internal density $\rho$) of mass $M$ and radius $R$. Show that the gravitational energy released during its formation is

$$E = \frac{3}{5} \frac{GM^2}{R}.$$  \hspace{1cm} (3)

13. The four long-lived radioactive isotopes which currently heat Earth’s interior are $^{238}\text{U}$, $^{235}\text{U}$, $^{232}\text{Th}$, and $^{40}\text{K}$. The table below gives the half-life $\tau_{1/2}$ of each isotope (in units of $10^9$ yr), and the power per unit mass of mantle rock $\varepsilon_{\text{now}}$ each isotope currently generates (in units of $10^{-12}$ W kg$^{-1}$ = $10^{-12}$ m$^2$ s$^{-3}$).

<table>
<thead>
<tr>
<th></th>
<th>$^{238}\text{U}$</th>
<th>$^{235}\text{U}$</th>
<th>$^{232}\text{Th}$</th>
<th>$^{40}\text{K}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_{1/2}$</td>
<td>4.5</td>
<td>0.70</td>
<td>14.0</td>
<td>1.25</td>
</tr>
<tr>
<td>$\varepsilon_{\text{now}}$</td>
<td>2.91</td>
<td>0.125</td>
<td>3.27</td>
<td>1.08</td>
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</tbody>
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(a) Which of these isotopes made the greatest contribution to Earth’s internal heating $4.5 \times 10^9$ yr ago?
(b) Calculate the total $\varepsilon$ at that time.

14. What would the ocean tides be like if the Earth was fixed in inertial space, and therefore prevented from accelerating even though subject to the Moon’s gravitational field?
(a) Describe the tides qualitatively.
(b) Estimate their height in meters.

15. Suppose the Moon’s mass doubled, with no immediate change in its present orbit.
(a) How much would the height of tides on Earth change as a result?
(b) How would the precession of the Moon’s orbit change as a result?
(c) How would the precession of the Earth’s rotation change as a result?
(d) How would the gradual expansion of the Moon’s orbit change as a result?
(e) How would the gradual decrease in the Earth’s rate of rotation change as a result?