Review the questions below, and be prepared to discuss them in class. For each question, list (a) the general topic, and (b) the key laws and equations needed to find an answer; bring this list to class on Nov. 12. Modified versions of some of these questions will be used in the midterm exam on Nov. 14.

1. The radial velocity method of planet finding yields an estimated planet mass $M_P \sin(i)$, where $i$ is the angle between the planet’s orbital angular momentum vector and our line of sight to the planet. (By definition, $0^\circ \leq i \leq 180^\circ$, so $\sin(i) \geq 0$.)
   (a) Assuming that the orbital angular momenta of other planetary systems point in random directions, $\cos(i)$ is uniformly distributed between $-1$ and $+1$. What is the probability that the inclination $i$ is between $60^\circ$ and $120^\circ$?
   (b) By what factor are planetary masses underestimated for inclinations between $60^\circ$ and $120^\circ$?

2. We observe planetary transits across a star with the same mass $M_\ast$ and radius $R_\ast$ as the Sun. The period between transits is $P = 10$ day, and during the transit the star appears to have 99% of its normal brightness. Assume the transiting planet has a circular orbit.
   (a) What is the radius of the planet?
   (b) What is the maximum amount of time a transit can last? Under what circumstances would we observe this maximum duration?
   (c) Suppose the transit is observed to last 2 hour (measured from the instant when the brightness falls to 99.5% to the instant when it returns to that level). What is the planet’s orbital inclination $i$?

3. Compare the angular momentum of the Sun’s rotation to the orbital angular momentum of the planets. Use $I = 0.073 M_\odot R_\odot^2$ for the Sun’s moment of inertia, and assume the Sun’s rotation period is 25 day. What fraction of the Solar System’s total angular momentum resides in the Sun? Which planet has the most angular momentum?

4. Compare the angular momentum of Jupiter’s rotation to the orbital angular momentum of the four Galilean moons. Use $I = 0.254 M_J R_J^2$ for the Jupiter’s moment of inertia. Jupiter and its four moons are sometimes called a “solar system in miniature”; explain how this analogy breaks down.

5. Consider a spherical cloud core of uniform initial density $\rho_0$. Assuming pressure forces and rotation are negligible, the cloud will simply collapse inward under its own gravity. Derive
an equation for the “free-fall” collapse time $t_{ff}$. (Hint: consider a particle on the edge of the cloud, and approximate its inward trajectory as a very elongated Keplerian ellipse. Then explain why the same collapse time applies throughout the cloud.)

6. Recall that a planet’s Hill radius $r_H$ is basically the radius at which the period of a satellite’s orbit around the planet is equal to the period of the planet’s orbital period around the Sun.
   (a) What is the Hill radius for the Earth in its orbit around the Sun?
   (b) In its present orbit, is the Moon in any danger of being stolen from us by the Sun?
   (c) At present, the Moon’s orbit accounts for $\sim 83\%$ of the total angular momentum in the Earth-Moon system. Eventually, tidal torques will transfer almost all of the remaining $\sim 17\%$ of the system’s angular momentum to the Moon’s orbit. When that happens, will the Moon still be within the Earth’s Hill radius?

7. The outer radius of the Oort cloud could be limited by two different effects: (i) the gravitational field of the entire galaxy, and (ii) the gravitational fields of passing stars and molecular clouds.
   (a) By analogy with the definition in problem 6, estimate the Sun’s Hill radius with respect to the galaxy. (Note: it takes the Sun $\sim 2.5 \times 10^8$ yr to orbit the galaxy.)
   (b) Given that the outer radius of the Oort cloud is probably about $10^5$ AU, is it likely that the cloud is actually limited by the entire galaxy, or by passing stars and molecular clouds instead?

8. Consider an asteroid which approaches a planet of mass $M$ and radius $R$ at initial velocity $v_0$ and impact parameter $b$, as shown in Fig. 1. If $v_0$ is very large, impacts only occur if $b \leq R$, but at lower velocities the planet’s gravity “focuses” asteroid trajectories, and impacts also occur if $R \leq b \leq b_{\text{max}}$. For a given $v_0$, what is the largest value of $b$ which results in an impact?

Figure 1: Orbit of an asteroid which just grazes a planet. Before being accelerated by the planet’s gravity, the asteroid’s initial velocity was $v_0$; if it had continued in a straight line, it would have passed the planet at minimum distance $b$. At the point of impact, it is moving parallel to the planet’s surface.
9. On Earth, you can probably jump 0.3 m off the ground. On a sufficiently small asteroid, the same jump would enable you to escape the asteroid’s gravity forever. Assuming this asteroid is spherical and has a density \( \rho \simeq 3000 \text{ kg m}^{-3} \), find the maximum radius such an asteroid can have. (You may ignore the weight of your space-suit; we’re talking about Le Petit Prince here.)

10. Bodies a and b, with masses \( M_a \) and \( M_b \), move in circular orbits about their mutual center of mass. Let \( r \) be the distance between the two bodies; their orbital period \( P \) is given by Newton’s version of Kepler’s third law. In the orbital plane, construct an equilateral triangle with one corner at the position of body a, and the second corner at body b. The third corner of the triangle is a Lagrange point; if this point lies ahead of body b then it’s the L₄ point, while if it lies behind then it’s the L₅ point.
   (a) At the L₄ point, show that the total gravitational force on a small mass \( m \), due to the two bodies a and b, points directly toward their mutual center of mass.
   (b) Show that the total magnitude of the gravitational force on \( m \) is exactly what’s required to keep it in a circular orbit around the center of mass with period \( P \). (This proves that L₄ is an equilibrium point; by symmetry, the same is true of L₅.)

11. Jupiter’s orbit around the Sun has a semi-major axis of \( a_J \simeq 5.2 \text{ AU} \). Asteroids which have orbital periods \( 1/4, 1/3, 2/5, 3/7, \) and \( 1/2 \) of Jupiter’s orbital period are in mean-motion resonances with Jupiter. Resonant effects increase the orbital eccentricities of such asteroids until they collide with other asteroids or planets, and consequently very few asteroids are found with these orbital periods. What semi-major axes do these five resonances have?

12. In Saturn’s rings, the 2 : 1 mean-motion resonance with Mimas creates the Cassini division between the A and B rings. In the asteroid belt, similar mean-motion resonances with Jupiter create the Kirkwood gaps, evident when one plots a histogram of asteroid semi-major axes \( a \). Yet no gaps are visible on a plot of asteroid positions; instead of displaying divisions at the radii corresponding to the Kirkwood gaps, the asteroids fill out a fairly uniform belt between \( \sim 1.8 \) and \( \sim 3.5 \text{ AU} \) from the Sun. Explain.

13. The Keeler gap in Saturn’s A ring (Fig. 2) is \( \Delta r_K \simeq 42 \text{ km} \) wide. Saturn’s moon Daphnis orbits in the middle of the Keeler gap; its orbital radius is \( r_d \simeq 1.37 \times 10^5 \text{ km} \). Saturn’s mass is \( M_S \simeq 5.7 \times 10^{26} \text{ kg} \).
   (a) Daphnis is roughly spherical, with radius \( R_d \simeq 4 \text{ km} \); assuming its density is \( \rho \simeq 1000 \text{ kg m}^{-3} \), find its mass \( M_d \).
   (b) Idealize the gravitational encounter of Daphnis with a particle at the inner edge of the outside ring as follows: Daphnis is fixed at the origin, and the ring particle moves by on a straight line at constant velocity \( v_p \simeq 1.28 \text{ m s}^{-1} \), missing Daphnis by 21 km at time \( t = 0 \). Integrate the acceleration of the ring particle from \( t = -\infty \) to \( t = +\infty \) to compute the inward velocity \( \Delta v_t \) the particle acquires as a result of its encounter with Daphnis. (Note:
Figure 2: The Keeler gap, with Daphnis. Orbital motion about Saturn is counter-clockwise; the planet is far below the bottom of the picture.

in reality, the particle will not move in a straight line, being deflected by Daphnis during the encounter; however, the straight-line model gives a good approximation of $\Delta v_r$.

(c) Using your result for $\Delta v_r$, estimate the radial (ie, inward and then outward) amplitude of the waves Daphnis creates. Compare your result to the amplitude seen in the photograph.

14. Asteroids which rotate too fast may break up because centrifugal forces overcome self-gravity. This problem contrasts two different estimates of the minimum rotation period for an asteroid composed of material with density $\rho \simeq 3000 \text{ kg m}^{-3}$.

(a) Assume the asteroid is spherical, and it’s covered by a layer of loose rock and gravel. What is the shortest rotation period this asteroid can have without losing material from its equator?

(b) Assume the asteroid consists of two solid spherical “lobes” which are held together by gravity and tumble end-over-end. What is the shortest rotation period this configuration can have?

(c) A few asteroids are observed to rotate faster than these limits. How is this possible?

15. Comets are deflected by jets of gas (primarily water vapor) escaping from sub-surface cavities (Fig. 3). The velocity of such a jet is roughly the sound speed

$$c_s = \sqrt{\frac{\gamma k_B T}{m}}, \quad (1)$$

where the adiabatic index for water vapour is $\gamma \simeq 1.3$, $k_B$ is Boltzmann’s constant, $T$ is the gas temperature, and $m$ is the mass of a gas particle.

(a) Evaluate $c_s$ for a typical gas temperature $T \simeq 200 \text{ K}$.

(b) At a distance of $r = 1 \text{ AU}$ from the Sun, a comet nucleus with a radius of $R = 1 \text{ km}$
loses mass at a rate of \( \dot{m} \simeq 1200 \text{ kg s}^{-1} \). Making the unrealistic assumption that all this material is channeled into a single jet with velocity \( c_s \), what is the resulting force on the comet nucleus?

(c) In principle, a comet nucleus is also subject to the Yarkovsky effect. The nucleus in part (b) is absorbing \( W \simeq 4 \times 10^9 \text{ kg m}^2 \text{ s}^{-3} \) of sunlight. Assuming that all this power is re-radiated as infrared photons in a specific direction, what is the resulting force on the comet nucleus?

16. Draw a phase diagram for hydrogen, including a sufficient range of pressures and temperatures to show the gas, liquid, solid, metallic, and ionized (plasma) phases. On this diagram, sketch a line showing the pressure and temperature you would encounter descending into Jupiter. Finally, discuss the physical state and characteristics of the matter in Jupiter, and explain why Jupiter has no solid surface.