

Chapter 6

Distances

6.1 Preliminaries

Techniques for measuring astronomical distances generally come in two varieties, *absolute* and *relative*.

Absolute distance measurements involve objects – possibly unique – with known intrinsic properties; their observed properties depend on distance D :

- **Known candles** have luminosity L . Assuming L is radiated isotropically, the flux received is $F = L/4\pi D^2$.
- **Known rulers** have size S . In the small-angle limit, the apparent angular size at distance D is $\theta = S/D$ radians.

Relative distance measurements involve *sets* of objects with standardized properties; inter-comparison of observed properties then yields distance ratios:

- **Standard candles** are sources which all have the same luminosity; if F_1 and F_2 are the observed fluxes from standard candles at distances D_1 and D_2 , then $D_1/D_2 = \sqrt{F_2/F_1}$.
- **Standard rulers** are objects which all have the same size; if θ_1 and θ_2 are the observed angular diameters of standard rulers at distances D_1 and D_2 , then $D_1/D_2 = \theta_2/\theta_1$.

Our knowledge of astronomical distances is built on a interconnected web of absolute and relative distance measurements. Relative measurements are used to build a “scale model” of the universe with correct proportions. Absolute measurements are used to determine the scale factor needed to express this model in physical units. In principle a single chain of relative measurements could fix the proportions of the model, and single absolute measurement could determine the scale. In practice, however, multiple absolute and relative distance measurements are used to guard against errors, insure consistency, and estimate uncertainties. This entire web of measurements is known as the cosmic **distance ladder**.

6.2 Distances to Stars

6.2.1 Geometric methods

Distances to stars near the Sun can be determined by **trigonometric parallax**. This uses the radius of the Earth's orbit, $1 \text{ AU} = 1.496 \times 10^{11} \text{ m}$, as a known ruler. Let $2p$ be the maximum shift in the apparent position of a nearby star as the Earth orbits the Sun (equivalently, $2p$ is the angular diameter of the Earth's orbit as seen from the star). Then the star's distance is $D = 1 \text{ AU}/p$ if p is expressed in radians, or $D = 206265 \text{ AU}(1''/p)$ if p is expressed in seconds of arc. The quantity $206265 \text{ AU} = 1 \text{ pc}$ is known as a **parsec**.

Ground-based astrometric techniques can measure p with errors of order $0.01''$, yielding distances good to $\pm 20\%$ at $\sim 20 \text{ pc}$. The Hipparcos satellite obtained parallaxes for $\sim 10^4$ fairly bright stars with errors of only $0.001''$, extending the scope of trigonometric parallax measurements to $\sim 200 \text{ pc}$.

To extend ground-based parallax measurements to greater distances requires the use of a baseline larger than the Earth's orbit. One option is to use the Sun's motion of $\sim 4 \text{ AU/yr}$ with respect to the standard of rest defined by stars in our neighborhood; for example, over an interval of 50 yr this yields a baseline of 200 AU , so the resulting **secular parallax** is 100 times the trigonometric parallax. The catch is that other stars also move with respect to the standard of rest, so there's no way to determine how much of an individual star's proper motion represents secular parallax. However, proper motions of an ensemble of stars distributed over the entire sky will be partly correlated due to the Sun's motion, and a analysis can determine the distance of stars in the ensemble. A related technique, known as **statistical parallax**, assumes that the radial velocities of an ensemble of stars are representative of their motions in three dimensions and uses proper motions to estimate distances; the details are complex, but the basic idea is simply that if some set of stars have similar random velocities with respect to the standard of rest, their average proper motions will be inversely proportional to their distances.

A final geometrical technique, of considerable historical importance, is the **moving cluster** method. In the simplest terms, consider a star cluster of fixed physical diameter which is receding at a known velocity v_{cl} . Let θ_1 and θ_2 be the cluster's angular diameters at times t_1 and t_2 . The distances to the cluster at these times have a ratio of $D_2/D_1 = \theta_1/\theta_2$ and a difference of $D_2 - D_1 = v(t_2 - t_1)$. Knowing the ratio and difference of the distances yields the distance at either time. This technique was used to estimate the distance to the Hyades cluster, with results of $D_{\text{Hyades}} \simeq (45 \pm 2) \text{ pc}$ and $(48 \pm 1) \text{ pc}$ in two different determinations. (In contrast, Hipparcos obtained a distance to the center of the Hyades of $\bar{D}_{\text{Hyades}} \simeq (46.3 \pm 0.3) \text{ pc}$.)

6.2.2 Photometric methods

To extend distance measurements beyond the maximum range of geometric methods we exploit the uniform photometric properties of stars. One powerful uniformity is provided by *main sequence* stars, which exhibit a well-defined relationship between luminosity and surface temperature. The relative distances of two star clusters can be estimated by comparing their color-magnitude diagrams; neglecting dust obscuration, the cluster whose main sequence appears δm magnitudes fainter is $\sim 10^{0.2\delta m}$ times further away. In practice, dust influences the observed colors and magnitudes, and

its effects must be independently determined, for example by use of *reddening-free indices* (BM98, Ch. 3.7.1), before reliable results can be obtained. Further uncertainties arise if the clusters compared have different metallicities or ages. Nonetheless, the technique of **main sequence fitting**, calibrated by absolute estimates of the distance to the Hyades, is important in constructing the distance ladder.

Pulsating *variable stars* were used to demonstrate for the first time the existence of galaxies beyond the Milky Way, and figure prominently in the most recent determinations of the Hubble constant. Such stars are valuable for distance measurements because they obey *period-luminosity* (PL) relations. The calibration of these relations is a critical problem; variable stars of a given period P may have a small range of luminosities, but unless the typical luminosities of these stars can somehow be determined by other means, they can only be used for relative distance estimates.

- (Classical) **Cepheid** variables are massive stars which become unstable after becoming red giants. They are intrinsically bright ($M_V \simeq -3$) and can thus be detected to distances of many Mpc. A few Cepheid variables are close enough for Hipparcos to obtain reliable distances, but these are insufficient to define an accurate PL relation. The LMC contains many Cepheids, and the uncertainty in the LMC's distance is presently the dominant source of uncertainty in the Cepheid PL relation.
- **RR Lyrae** variables are old metal-poor horizontal branch stars. They are less luminous than Cepheids ($M_V \simeq 0.6$), and thus too faint to detect beyond the Local Group. RR Lyrae stars are found in globular clusters and elsewhere in the bulge and halo of the Milky Way; some are also found among old, metal-poor disk stars.
- **Mira** variables are AGB stars – relatively low-mass stars which have evolved to the point where they are burning helium in a shell around an inert carbon/oxygen core. They have large amplitudes and periods longer than ~ 100 day. In the K band they exhibit a well-defined period-luminosity relationship (Feast et al. 1989).

Independent estimates of distances to variable stars are needed to calibrate PL relationships. In addition to trigonometric parallaxes, statistical parallaxes, and main-sequence fitting (for variables in star clusters), the absolute distance to a pulsating variable star can be estimated by the **Baade-Wesselink** method. This technique employs high-resolution spectroscopy to measure the variation of a star's radial velocity as it expands and contracts. The time integral of the velocity gives the absolute change in the star's radius from minimum to maximum, $R_{\max} - R_{\min}$; meanwhile, the variation in observed flux together with the Stefan-Boltzmann law gives the ratio of the star's radii, R_{\max}/R_{\min} . These quantities can be combined with surface temperature data to determine the star's radius and hence its absolute luminosity.

Observations of **eclipsing spectroscopic binary stars** can provide relatively clean absolute distance measurements. The main idea is to determine the star's orbital velocities from the periodic shifts in their spectral lines; with the observed durations of eclipses, it's possible to solve for the stars' radii. Knowledge of radii and surface temperatures then yield absolute luminosities.

6.3 Distances to Galaxies

Galaxies close enough to be resolved into individual stars are often good candidates for distance determinations using the photometric methods outlined above. For more distant galaxies, a variety

of relative methods exist; these are calibrated by comparison with nearby galaxies.

6.3.1 Luminosity functions

Populations of various objects in galaxies – globular clusters, planetary nebulae, and stars – have luminosity functions which are relatively independent of local conditions. When a large sample of these objects can be observed, their apparent brightnesses may be analyzed to obtain an estimate for the distance of their parent galaxy:

- **Globular cluster luminosity functions** are Gaussian in M :

$$\phi_{\text{GC}}(M) \propto \exp \left[-\frac{(M - \bar{M}_{\text{GC}})^2}{2\sigma_{\text{GC}}^2} \right] \quad (6.1)$$

In the Milky Way, $\bar{M}_{\text{GC}} \simeq -7.4 \pm 0.1$ in the V band, and globulars in other galaxies *appear* to be drawn from the same luminosity function. In practice, the census of globular clusters in a galaxy must be complete well below the peak of the luminosity function to obtain a reliable distance.

- **Planetary nebulae luminosity functions** cut off sharply at the bright end: $M_{\text{PN}}^{\text{cut}} \simeq -4.6 \pm 0.1$ in M31 (assuming a distance of 740 kpc).
- **Surface-brightness fluctuations** in early-type galaxies depend on the stellar luminosity function. Suppose we observe two galaxies at distances $D_1 < D_2$ with the same surface brightness. The average number of stars per resolution element N scales as the square of the distance, so $N_2/N_1 = (D_2/D_1)^2$. The actual number of stars per resolution element exhibits \sqrt{N} fluctuations about the average value N , creating a slightly mottled appearance. For either galaxy the variance in the flux is $\sigma_F \simeq \sqrt{N}/N = 1/\sqrt{N}$, so the more distant galaxy appears smoother than its nearby counterpart: $\sigma_{F,2}/\sigma_{F,1} = D_1/D_2$. This method is most reliable in the IR where variations in the intrinsic luminosity function from galaxy to galaxy are not so significant.
- The **tip of the red giant branch** has a well-defined maximum brightness. A star's path up the red giant branch (RGB) ends when helium ignites in its core, and for RGB stars with ages over 2 Gyr this event occurs at a specific bolometric luminosity. In the I band, this corresponds to absolute magnitude

$$M_{\text{TRGB}} = -3.629 + 0.48[\text{Fe}/\text{H}] + 0.14[\text{Fe}/\text{H}]^2, \quad (6.2)$$

where the dependence on metallicity $[\text{Fe}/\text{H}]$ is largely due to line-blanketing in stellar atmospheres (Bellazzini et al. 2004).

6.3.2 Kinematic indicators

Photometric and kinematic properties of galaxies are often related; since kinematic parameters are distance-independent, these relations can be calibrated to determine distances.

- The **Tully-Fisher relation** (Tully & Fisher 1977) connects the rotation velocity of a spiral galaxy to its absolute luminosity. This relationship is conveniently expressed in terms of

the width W of a galaxy's integrated velocity profile; after correcting for inclination and the contribution of random velocities, the galaxy's I-band absolute magnitude is

$$M_I = -8.72(\log W - 2.50) - 20.94 \pm 0.10 \quad (6.3)$$

(Pierce & Tully 1992).

- The **Faber-Jackson relation** (Faber & Jackson 1976) connects the central velocity dispersion of an elliptical galaxy to its total luminosity; see (4.14). In practice, the intrinsic scatter in this relationship is rather large, so it's of limited use in determining distances.
- The **fundamental plane** of elliptical galaxies introduces a third parameter correlated with the scatter in the Faber-Jackson relation. For example, (4.15) expresses the effective radius R_e in terms of the central velocity dispersion σ and the surface brightness at the effective radius I_e ; both σ and I_e are distance-independent, so this relationship may be used to calculate R_e . A slight variant is the $D_n - \sigma$ relation; the diameter D_n within which the mean surface brightness is $20.75 \text{ mag}/''$ in B is empirically

$$D_n = 2.05 \text{ kpc} \left(\frac{\sigma}{100 \text{ km/s}} \right)^{1.33}. \quad (6.4)$$

The calibration of these indicators requires reasonably large samples of suitable galaxies with independently determined distances. For the Tully-Fisher relation, Cepheid variables are often used to obtain the necessary distances. Cepheids are not found in elliptical galaxies, so the calibration of the $D_n - \sigma$ relation rests on a more heterogeneous set of distance determinations. However, elliptical galaxies occur in clusters, and the distance to a cluster can be determined more precisely by averaging estimated distances to individual cluster members.

6.3.3 Supernovae

Type Ia Supernovae (Sn Ia) are thought to occur when mass accretion causes a white dwarf star to exceed the Chandrasekhar limit of $1.4 M_\odot$. As such, they are basically "standard bombs" with properties such as peak absolute magnitude M_{peak} very nearly independent of local conditions. In practice things are more complex; M_{peak} is correlated with the rate of decline in a manner which must be empirically calibrated. Moreover, dust extinction must be determined by simultaneous observations at several wavelengths. Nonetheless, SN Ia are so bright that they are visible to redshifts $z > 1$, and distances determined in this way are crucial for cosmological studies, including recent evidence that the expansion of the universe is accelerating.