

Chapter 15

Kinematics of the Solar Neighborhood

Unlike an elliptical galaxy, the Milky Way rotates with a speed much larger than the random velocities of typical stars. Our position inside the disk of the Milky Way is both a blessing and a curse; we can obtain very detailed data on stellar motions, but must take our own motion into account when interpreting the observations.

15.1 Standards of Rest

Motions in the Milky Way are commonly expressed with respect to the

- **FSR**: the ‘fundamental’ standard of rest with respect to the galactic center, or
- **LSR**: the ‘local’ standard of rest with respect to a circular orbit at the Sun’s radius.

The FSR is used when describing the MW as a whole, while the LSR is more useful for describing motions near the Sun.

Since the MW is approximately rotationally symmetric, we adopt a cylindrical coordinate system; let z be distance above the plane of the MW, R be the distance from the galactic center in the plane of the MW, and ϕ be the azimuthal coordinate, measured in the direction of the MW’s rotation.

The LSR moves about the galactic center at the local value of the circular velocity. Velocities with respect to the LSR are represented by vertical, radial, and azimuthal components

$$w \equiv \frac{dz}{dt}, \quad u \equiv -\frac{dR}{dt}, \quad v \equiv R \frac{d\phi}{dt} - V(R_0), \quad (15.1)$$

respectively, where $V(R)$ is the circular velocity at radius R , and R_0 is the Sun’s galactic radius. Thus a star with $w > 0$ is climbing above the galactic plane; a star with $u > 0$ is falling inward toward the galactic center, and a star with $v > 0$ is moving in the direction of galactic rotation at greater than the local circular velocity.

The determination of the LSR is a two-step process. Basically, we assume that *on average* stars in the Sun’s vicinity are moving in the ϕ direction but have no net motion in the R or z directions. It might also seem natural to assume that on average the stars are also moving in the ϕ direction with

the local circular velocity, but this is not true because of the *asymmetric drift* (see below); the greater the random velocities of the stars, the more the net motion lags behind the local circular velocity. Thus the steps are

1. measure the solar motion with respect to an ensemble of nearby stars selected in a kinematically *unbiased* manner, and
2. correct for the asymmetric drift due to random motions of stars.

Step #2 may be performed empirically by investigating how the asymmetric drift velocity v_a depends on the radial velocity dispersion $\overline{u^2}$ for different sets of stars, and extrapolating to $\overline{u^2} = 0$. The data are well-fit by a linear relation:

$$v_a \simeq \frac{\overline{u^2}}{80 \pm 5 \text{ km s}^{-1}} \quad (15.2)$$

(BM98, eq. (10.12)). Results for different ensembles of stars may be combined to obtain the motion of the Sun with respect to the LSR:

$$(u, v, w)_\odot = (10.0 \pm 0.4, 5.2 \pm 0.6, 7.2 \pm 0.4) \text{ km s}^{-1} \quad (15.3)$$

(BM98, eq. (10.11)).

To relate the LSR to the FSR, we need to know the local circular velocity, $V_0 = V(R_0)$. In 1985, IAU Commission 33 adopted $V_0 = 220 \text{ km s}^{-1}$ and $R_0 = 8.5 \text{ kpc}$ (Kerr & Lynden-Bell 1986). More recent estimates favor slightly smaller values of R_0 ; BM98 adopt $R_0 = 8.0 \text{ kpc}$.

15.2 Effects of Galactic Rotation

The rotation of the galaxy gives rise to an organized pattern of stellar motions in the vicinity of the Sun. In this section, these motions are described under the assumption that random velocities are zero.

15.2.1 Intuitive picture

A physical understanding of the kinematic consequences of galactic rotation may be gained by considering separately the local effects of *solid-body* and *differential* rotation (MB81, Chapter 8-1).

If the MW rotated as a solid body, with angular velocity ω independent of R , then distances between stars would not change, and all radial velocities would be zero. However, stars would still show proper motions with respect to an external frame of reference; the transverse velocity of a star at a distance r from the Sun would be

$$v_t = -\omega r. \quad (15.4)$$

Of course, the MW does *not* rotate as a solid body; the orbital *period* is an increasing function of R in the vicinity of the Sun. Stars at radii $R < R_0$ therefore catch up with and pass us, while we catch up with and pass stars at radii $R > R_0$. This results in non-zero radial velocities,

$$v_r \propto r \sin(2\ell), \quad (15.5)$$

where ℓ is the galactic longitude of the star under observation.

15.2.2 Global formulae

Consider a star at galactic radius R moving at the circular velocity appropriate to that radius, $V(R) = R\omega(R)$. With respect to the LSR at the Sun's galactic radius R_0 , the radial and transverse components of the star's motion are

$$v_r = (\omega - \omega_0)R_0 \sin(\ell), \quad v_t = (\omega - \omega_0)R_0 \cos(\ell) - \omega r, \quad (15.6)$$

where $\omega_0 = \omega(R_0)$.

15.2.3 Local approximations

In the local limit, where r/R_0 is a small parameter, the above expressions become

$$v_r \simeq Ar \sin(2\ell), \quad v_t \simeq (A \cos(2\ell) + B)r, \quad (15.7)$$

where A and B are the Oort constants, given by

$$A \equiv \frac{1}{2} \left(\frac{V}{R} - \frac{dV}{dR} \right)_0, \quad B \equiv -\frac{1}{2} \left(\frac{V}{R} + \frac{dV}{dR} \right)_0, \quad (15.8)$$

where the subscript 0 indicates that the expressions in parenthesis are evaluated at the solar radius, R_0 . In brief, A is a measure of the shear of the MW's rotation, while B is a measure of the vorticity.

Observations of local stellar motions allow a direct estimate of the Oort constants. In practice these quantities are subject to a number of constraints; a recent determination from Hipparcos data yields

$$A \simeq 14.8 \pm 0.8 \text{ km s}^{-1} \text{ kpc}^{-1}, \quad B \simeq -12.4 \pm 0.6 \text{ km s}^{-1} \text{ kpc}^{-1} \quad (15.9)$$

(Feast & Whitelock 1997).

15.3 Random Velocities in the Solar Neighborhood

Like the Sun, other stars have random velocities with respect to the LSR. Complementing the discussion above, this section will discuss random motions of stars in our immediate neighborhood, while neglecting the larger-scale effects of rotation.

15.3.1 Theoretical expectations

Consider an ensemble of stars with orbits passing through the vicinity of the Sun. Since the Milky Way is about 50 rotation periods old, it is reasonable to assume that stars are well-mixed; that is, slight differences in orbital period will have had enough time to spread out initially-correlated groups of stars, in effect assigning stellar orbits randomly-chosen phases. In an axisymmetric galaxy, the velocity ellipsoid at $z = 0$ should then have principal axes aligned with the R , ϕ , and z directions.

To a first approximation, histograms of random stellar velocities do not differ much from gaussians, so a convenient approximation to the velocity distribution with respect to the LSR of a well-mixed stellar ensemble is

$$f(u, v, w) = f_0 \exp(-Q(u, v - v_a, w)), \quad (15.10)$$

where v_a is the asymmetric drift velocity of the ensemble, the function

$$Q(u, v', w) = (u, v', w) \cdot \mathbf{T} \cdot (u, v', w), \quad (15.11)$$

and the symmetric tensor

$$\mathbf{T} \equiv \frac{1}{2} \begin{bmatrix} \sigma_R^2 & 0 & 0 \\ 0 & \sigma_\phi^2 & 0 \\ 0 & 0 & \sigma_z^2 \end{bmatrix} \quad (15.12)$$

describes the shape of the velocity ellipsoid. Note that \mathbf{T} is proportional to the *random* part of the kinetic energy tensor of the stellar ensemble; it is diagonalized as a consequence of the mixing assumption made above. For this \mathbf{T} , the quantity $Q(u, v - v_a, w)$ is

$$Q = \frac{1}{2} \left(\frac{u^2}{\sigma_R^2} + \frac{(v - v_a)^2}{\sigma_\phi^2} + \frac{w^2}{\sigma_z^2} \right). \quad (15.13)$$

15.3.2 Observational results

In practice the velocity ellipsoid for a given ensemble of stars (*e.g.* all stars of a given stellar type) is never perfectly diagonalized. The most significant term to be added to (15.13) is proportional to $u(v - v_a)$, indicating that the velocity ellipsoid lies in the plane of the disk, but is not precisely oriented toward the galactic center. The angle between the long axis of the velocity ellipsoid and the R direction is called the *longitude of vertex*, ℓv .

Results for a wide range of stellar types are listed in Tables 10.2 and 10.3 of BM98. For dwarf stars, the various components of the velocity dispersion become greater progressing from early to late spectral types. This is an age effect: late spectral classes include more old stars, and the random velocities of stars increase with time (presumably due to gravitational scattering, although a complete theory is not available). For giant stars, kinematic parameters reflect those of the dwarf stars they evolved from. The largest random velocities are found in samples of white dwarfs which include the largest fractions of very old (age $\sim 10^{10}$ year) stars.

The angle ℓv depends on the sample of stars studied, ranging from $\sim 30^\circ$ for the earliest main-sequence samples to $\sim 10^\circ$ for later samples. This trend with spectral type is probably an age effect; older stars are more completely mixed. But even the latest samples still show a significant deviation; spiral structure in the galactic disk may be to blame.

While the overall distribution of random velocities is roughly consistent with (15.10), significant structures occur in the stellar velocity distribution. These ‘star streams’ or ‘moving groups’ are interpreted as the remains of stellar associations and clusters which have been sheared out by the tidal field of the Milky Way.

15.4 Asymmetric Drift

Empirically, the net lag of a given ensemble of stars with respect to the LSR is approximated by (15.2) above. This relationship is a consequence of the collisionless Boltzmann equation. In cylindrical coordinates, the CBE for a steady-state axisymmetric system is (*e.g.* BT87, Chapter 4.1a)

$$0 = v_R \frac{\partial f}{\partial R} + v_z \frac{\partial f}{\partial z} + \left(\frac{v_\phi^2}{R} - \frac{\partial \Phi}{\partial R} \right) \frac{\partial f}{\partial v_R} - \frac{v_R v_\phi}{R} \frac{\partial f}{\partial v_\phi} - \frac{\partial \Phi}{\partial z} \frac{\partial f}{\partial v_z}. \quad (15.14)$$

To derive the relationship for the asymmetric drift we take the radial velocity moment of (15.14); multiplying by v_R and integrating over all velocities, the result is a Jeans equation:

$$0 = \frac{R}{v} \frac{\partial}{\partial R} (v \overline{v_R^2}) + R \frac{\partial}{\partial z} (\overline{v_R v_z}) + \overline{v_R^2} - \overline{v_\phi^2} + R \frac{\partial \Phi}{\partial R}. \quad (15.15)$$

The azimuthal motion can be divided into net streaming and random components:

$$\overline{v_\phi^2} = \overline{v_\phi}^2 + \sigma_\phi^2 = (V_0 - v_a)^2 + \sigma_\phi^2, \quad (15.16)$$

where the second equality follows from the definition of the asymmetric drift velocity v_a . For the radial motion,

$$\overline{v_R^2} = \sigma_R^2, \quad (15.17)$$

since there is no net streaming motion in the radial direction. Combining (15.15), (15.16), and (15.17), using the identity $V_0^2 = R d\Phi/dR$, and assuming that v_a/V_0 is small, we obtain

$$v_a \simeq \frac{\sigma_R^2}{2V_0} \left[\frac{\sigma_\phi^2}{\sigma_R^2} - 1 - \frac{\partial \ln}{\partial \ln R} (v \sigma_R^2) - \frac{R}{\sigma_R^2} \frac{\partial}{\partial z} \overline{v_R v_z} \right]. \quad (15.18)$$

This equation relates the asymmetric drift velocity to the radial component of the velocity dispersion. If we compare ensembles of stars which have the same radial distribution and velocity ellipsoids of similar shapes, the expression inside square brackets is constant and we recover (15.2).

Several of the terms in (15.18) may be further simplified. By multiplying the CBE by $v_R v_\phi$ and integrating over all velocities it is possible to show that

$$\frac{\sigma_\phi^2}{\sigma_R^2} = \frac{-B}{A-B}, \quad (15.19)$$

(BT87, Ch. 4.2.1(c)). Here A and B are the Oort constants; adopting the values and uncertainties in (15.9), we expect $\sigma_\phi^2/\sigma_R^2 \simeq 0.46 \pm 0.05$. This is reasonably consistent with observational data, which give $\sigma_\phi^2/\sigma_R^2 \simeq 0.3$ to 0.5 (BM98, Ch. 10.3.2).

The most uncertain part of (15.18) is the last term within the square brackets. This term represents the tilt of the velocity ellipsoid at points above (and below) the galactic midplane, $z = 0$. We do not presently know very much about the orientation of the velocity ellipsoid away from the midplane. If the ellipsoid remains parallel for all z values then this term is identically zero, while if the ellipsoid tilts to always point at the galactic center then

$$\frac{R}{\sigma_R^2} \frac{\partial}{\partial z} \overline{v_R v_z} = 1 - \frac{\sigma_z^2}{\sigma_R^2}. \quad (15.20)$$

Numerical experiments indicate that the most likely behavior is somewhere between these two extremes.

