Some Uses of Astrometry

• stacking of images not pointed to the exact same location on the sky
• identification of optical counterparts to radio sources
• parallax and proper motion of nearby stars
• orbits of asteroids and comets
Reference Catalogs

• Bonner/Cordoba Durchmusterung
  Bonner north of $-22$ deg
down to about magnitude 9
epoch 1855

• FK3/FK4/FK5
  1535 fundamental stars
down to about magnitude 7
FK3 adopted in 1935
FK5 adopted in 1988
More Reference Catalogs

- **Yale Bright Star Catalogue**
  2nd Edition 1940
  3rd Edition 1964
  9091 stars
  down to magnitude 6.5
  1900 and 2000 epochs

- **SAO (1966)**
  258997 stars
  down to about magnitude 9
  1950 epoch
More Reference Catalogs

• AGK3 (1975)  
down to about magnitude 9  
1950 epoch

• PPM (1993)  
378901 stars  
down to about magnitude 9  
2000 epoch
More Reference Catalogs

• HST Guide Star Catalog
down to magnitude 16

• Hipparcos (1989-1993)
  118218 stars
  limiting magnitude 12.4
  median precision 0.7 milliarcsec

• Tycho
  1058332 stars
  limiting magnitude 11.5
  median precision 25 milliarcsec
More Reference Catalogs

- **UCAC2 (2003)**
  - 20 cm astrograph, 1 deg field
  - 4k CCD, 0.9 arcsec pixels
  - 48,330,571 stars
  - Declinations $-90$ to $+40$ (some $+52$)
  - R magnitudes 7.5 to 16
  - 20/70 milliarcsec (mag 10-14/16)
  - 2000 epoch
  - ICRS reference frame
  - Whole sky catalog in 2005
More Reference Catalogs

• **USNO-A1.0**
  
  all sky
  limiting magnitude about 20
  no proper motions
  epoch 2000
  precision around 0.3 arcsec

• **USNO-SA**

  subset of the A catalog
  uniform distribution of stars
More Reference Catalogs

• **USNO-A2.0**
  500 million sources
  12 bytes per source
  no proper motions

• **USNO-B1.0 (2003)**
  1 billion sources
  80 bytes per source
  proper motions
  precision around 0.1 arcsec
  lots of bogus doubles
Tangent Plane Projection

\[ \xi = \frac{\cos \delta \sin (\alpha - A)}{H} \]
\[ \eta = \frac{[\sin \delta \cos D - \cos \delta \sin D \cos (\alpha - A)]}{H} \]

where

\[ H = \sin \delta \sin D + \cos \delta \cos D \cos (\alpha - A) \]

and \( \alpha, \delta \) are coordinates of source and \( A, D \) are coordinates of tangent point.
Reverse Procedure

\[ \tan (\alpha - A) = \frac{\xi}{\Delta} \]
\[ \tan \delta = \frac{\sin D + \eta \cos D}{\Gamma} \]

where

\[ \Delta = \cos D - \eta \sin D \]
\[ \Gamma = (\xi^2 + \Delta^2)^{1/2} \]

Orbit Computation

LaPlace’s method attempts to solve the differential equation

\[ r'' = -\frac{r}{r^3} \]

by writing the solution in the form of a Taylor’s series
Orbit Computation

Gauss’ method requires three pairs of RA-Dec values

\[ \mathbf{r}_2 = c_1 \mathbf{r}_1 + c_3 \mathbf{r}_3 \]

position vector at \( t_2 \) is a linear combination of position vectors at \( t_1 \) and \( t_3 \)

some cross products yield areas of the triangles

Kepler’s second law gives areas of sectors

attempts to solve the sector-triangle ratios
Orbit Computation

if one wishes to assume a circular orbit, then the eccentricity is zero and the argument of perihelion is irrelevant, leaving four unknowns, but Tisserand found that difficulties arise for short time intervals, proximity to opposition, and fast motions

method of Olbers often applied to newly discovered comets, for which the eccentricity is assumed to be unity, leaving five unknowns
Orbit Computation

These orbit determination methods are focused on finding a single set of classical orbital elements that satisfy the available observations, but the methods can fail under a variety of circumstances, particularly when the departure from great circle motion is negligible, which is almost always the case for very short observational arcs.

Especially at smaller solar elongations, double solutions are possible, and it is easy for a method that tries to find a single solution to get the wrong one.
Orbit Computation

Classical orbital elements
need one parameter to indicate size of the orbit
\( a = \) semimajor axis, or \( q = \) perihelion distance
need one parameter to indicate shape of the orbit
\( e = \) eccentricity
need three parameters to indicate orientation
\( i = \) inclination, \( \Omega = \) long. of ascending node,
\( \omega = \) argument of perihelion
need one parameter to indicate location in orbit
\( T = \) time of perihelion passage
Orbit Computation

these six parameters are not unique; could just as easily use heliocentric rectangular position and velocity vectors for a given instant of time

\[ x, y, z, x', y', z' \]

we choose to use a different set of six parameters, namely the topocentric spherical coordinates and their time derivatives for a given instant of time

\[ \alpha, \delta, \Delta, \alpha', \delta', \Delta' \]
Orbit Computation

The advantage of using these six quantities is that two of them, $\alpha$ and $\delta$, can be known to a precision of one part in a million after just one observation good to 1 arcsec; why struggle to find values for six poorly constrained parameters when two can be strongly constrained by a single observation?

A second observation can constrain $\alpha'$ and $\delta'$ to perhaps one part in a hundred, or better, depending on the total motion and the relative astrometric accuracy.
Orbit Computation

So, we choose to work through a grid of assumed values for the two remaining unknowns, $\Delta$ and $\Delta'$, using the unlikelihood of hyperbolic and retrograde orbits to constrain the allowed values. As more observations are added, one can then use statistical arguments about the quality of the fit to the observations to constrain the values further.

Advantages include the ability to work with very short arcs, great circle motion, and recognition of double solutions.
These ideas have been implemented in an orbit computation program called KNOBS. This generalized program is the successor to TWOOBS, which did the orbit computations for cases with just TWO OBServations, hence the name. The name KNOBS comes from N OBServations, but everybody knows that knobs is spelled with a k!