Fourier Analysis of Hubble Space Telescope Fine Guidance Sensor Data of Binary Stars and Application to the Multiple System HD 157948

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ABSTRACT

A Fourier-based method is presented for the analysis of binary and multiple star data taken with the Hubble Space Telescope Fine Guidance Sensors. Relative astrometry and magnitude differences are obtained as with standard FGS analysis techniques, and although the Fine Guidance Sensor system is essentially unfiltered, this method also permits the characterization of color differences between components of binary or multiple star systems based on the wavelength dependence of the interference fringes produced by the instrument. Using the multiple system HD 157948, we show that the method produces astrometric and photometric measurements that are consistent with previous FGS analysis for the three components that lie within the field of view of FGS and gives color differences relative to the primary for two of the three known companions. Speckle observations of the system with the WIYN 3.5-m Telescope at Kitt Peak National Observatory are also presented which, in combination with the FGS results

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and other data available in the literature, permit $B - V$ colors and absolute $V$ magnitudes to be obtained of all four components in the system. Their colors and magnitudes are plotted on the H-R diagram and a comparison with theoretical isochrones indicates that the results are consistent with theory for a relatively young, somewhat metal-poor system.

Subject headings: binaries: visual — binaries: spectroscopic — stars: individual (HD 157948) — techniques: interferometric — techniques: astrometric — techniques: photometric

1. Introduction

The Fine Guidance Sensors (FGSs) aboard the Hubble Space Telescope are a system that can be used either for telescope pointing or precision astrometry in two modes: position (POS) mode, and transfer function (TRANS) mode. In TRANS mode, relative astrometry of two or more objects that lie close together on the sky is obtained from two orthogonal transfer functions. These are determined from the interference patterns produced by the instrument, the heart of which is a Koesters prism that produces white-light interference fringes in the two arms of the interferometer. Two orthogonally positioned interferometers constitute a single FGS, and three FGSs are on board the telescope. For POS and TRANS mode data (i.e. science observations), only one of the three FGSs is available, the so-called FGS1r. (For more information on the instrument, see http://www.stsci.edu/hst/fgs).

TRANS mode observations are extremely well-suited to the observation of close binary stars. For each axis of the FGS, two phototubes record the amount of light obtained as a function of position in each arm of the 1-dimensional interferometer. If $A(x)$ represents the number of photons obtained in arm A as a function of position $x$ and $B(x)$ represents the number of photons obtained in arm B, then the transfer function is formed as follows:

\[
T(x) = \frac{A(x) - B(x)}{A(x) + B(x)}.
\]  

(1)

For a point source, the transfer function has a zero crossing at the location of the source, a maximum to one side of this location and a minimum to the other side, giving rise to the term “S-curve” when referring to the transfer function of the FGS. For a binary star, two such S-curves occur in the data, with location and relative height determined by the projected separation and magnitude difference of the two point sources.

The standard technique of analyzing FGS data involves the selection of a template S-curve, and then an iterative fitting procedure to determine both the height and location of each point source (e.g. Franz et al. 1991, 1992; Henry et al. 1999). The FGS1r does not have narrow filters and is
essentially pan-chromatic through the visible, although the quantum efficiency of the phototubes decreases at redder wavelengths. Color measurements have therefore not been common to date with the system. In addition, the morphology of the transfer function is dependent on color, so that, depending on the color difference between the components, a different template may be used for primary and secondary. The FGS Instrument Handbook (Nelan et al. 2004) states that the target and template should match to within 0.3 magnitudes in \( B - V \) color, but in general the color of the template used is not considered definitive for that of the source. This is because the number of templates that exist is very limited, and until recently there has only been a very rough color calibration due to effects such as the variation over time of the transfer function morphology with telescope focus and other detector-related effects. These effects appear to have diminished over time, as discussed in Nelan et al. (2004).

In 2001, we began a program of FGS observations of thirteen metal-poor, high proper motion spectroscopic binary stars from the sample of Carney and Latham (1987) to attempt to resolve these stars for the first time. All objects in the sample have distance determinations from the Hipparcos Catalogue (ESA, 1997). The main goal of the work is to characterize the metal-poor mass-luminosity relation, since a measure of the scale of the orbit (in addition to the Hipparcos distance measure for all single-lined systems) would lead directly to individual mass determinations. However, the observations have proved to be challenging to reduce with the standard analysis techniques since the separations and magnitude differences of our systems are often near the detection limits of FGS, and we became interested in the development of a complementary technique for the analysis of these data. One of us (E. H.) had previously developed a data reduction pipeline for speckle interferometry data of close binary stars in the Fourier domain which has provided excellent astrometric (e.g. Horch et al. 2002) as well as photometric (Horch, Meyer, and van Altena 2004) information of speckle binaries, and these routines were readily adaptable for use with FGS data.

We have therefore developed a reduction method based on the Fourier analysis of the transfer function that fits the data for the Fourier signature expected for two or more point sources, and our strategy has been to analyze our data set with both the standard and Fourier reduction techniques independently, and to require consistency between the methods before reporting the final results of the project. The development of the new technique also presents an opportunity to search and fit for the case where one component of a binary system differs in color from the other. One expects that the width of the S-curve is wavelength dependent, and therefore it is in theory possible to correlate this with the color of the source. In addition to the relative astrometry and magnitude difference in the wide FGS pass band, the method could then yield color difference estimates between components. There has been work in the past to study the transfer function in the Fourier domain, most notably by Hershey (1992) and later by Robinson et al. (2002), but these investigations have not addressed the measurement of color. We present the Fourier method and model results in the first part of the paper.

In the second part of the paper, we use the method in combination with speckle data and absolute photometry available in the literature to study the quadruple system HD 157948 = HIP
85209 = G 182-7. The innermost pair of this system is a double-lined spectroscopic binary, the orbit of which was determined by Latham et al. (1992) and has since been updated by Goldberg et al. (2002). The object is part of the Carney and Latham survey of high-velocity stars, and has metallicity measures of [Fe/H] = -0.76 (according to Latham et al.) and -0.5 (according to Goldberg et al.). The Hipparcos Catalogue (ESA, 1997) notes a wide component with \( \Delta V \approx 4.0 \) at a separation of approximately 2 arcseconds from the spectroscopic pair, originally discovered visually by Couteau (Couteau, 1975). We have successfully resolved the spectroscopic pair for the first time using FGS (first reported by Heasley et al. 2002). Heasley et al. also note a companion with separation 0.26 arcseconds from the spectroscopic pair which was apparently missed by Hipparcos. This component has been confirmed by one of us (E. H.) using the RYTSI speckle camera at the WIYN\(^3\) Telescope at Kitt Peak National Observatory. The combination of these data permits the estimation of component colors and magnitudes, as well as a preliminary visual orbit and masses for the spectroscopic pair. A comparison with the Yale-Yonsei isochrones is made, and we discuss the results of this comparison.

2. FGS Reduction Method

2.1. The Color-Difference Signature in the Fourier Domain

The Koesters interferometers inside FGS create a fringe pattern where the width between fringes is in principle determined by the wavelength of light coming into the instrument. However, since all colors through the visible range are passed through the instrument, the amplitude of the fringes dies away quickly as one moves from the location of the source, resulting in the S-curve previously mentioned. Nonetheless, one could in principle determine colors from a careful measurement of the width of the S-curve, if color were the sole factor in the width of this function.

For two sources within the FGS TRANS field of view, to first order telescope focus and other optical effects that influence the detailed shape of the S-curve will affect both primary and secondary transfer function widths in the same way. If the two sources have significantly different color, the resulting transfer function may still contain color difference information, even if the absolute width is difficult to interpret. To investigate this possibility, we have constructed a program that produces simulated FGS data. The input information is the spectrum of a star, from which the white-light fringes are constructed according to the phase differences obtained at each wavelength as a function of position. Input spectra from the empirical catalogue of Pickles (1998) are used in the case that real stellar spectra are desired, and the program will also accept blackbody spectra. Let the phase difference, \( \Delta \phi_A \), of the interfering beams along the exit face A of the Koesters prism be defined by

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\(^3\)The WIYN Observatory is a joint facility of the University of Wisconsin-Madison, Indiana University, Yale University, and the National Optical Astronomy Observatories.
\[ \Delta \phi_A = \frac{C'x'}{\lambda} \cdot (x - x_0), \]  

(2)

where the object is located at position \( x_0 \) on the sky, \( x \) represents the position on the sky corresponding to zero tilt relative to the instrument, \( x' \) is the coordinate along the exit face of the Koesters prism parallel to the \( x \)-axis of the instrument, \( \lambda \) is the wavelength, and \( C \) is a scale conversion factor. Then, one photomultiplier will receive a signal \( A \) such that

\[ A(x) \propto \int \int dx'd\lambda S(\lambda, x')F(\lambda) \cos^2\left(\frac{\pi}{4} + \Delta \phi_A\right) \]  

where \( S(\lambda, x') \) is a sensitivity function that incorporates the filter transmission, detector quantum efficiency, geometrical factors that arise from the shape of the collimated beam as it crosses the input face of the Koesters prism as well as any optical losses in the telescope or transfer optics, and \( F(\lambda) \) is the photon flux from the source at wavelength \( \lambda \). The other arm of the interferometer will receive signal \( B \) such that

\[ B(x) \propto \int \int dx'd\lambda S(\lambda, x')F(\lambda) \cos^2\left(\frac{\pi}{4} - \Delta \phi_B\right), \]  

(4)

where \( \Delta \phi_B \) is the corresponding phase difference function for output face B of the Koesters prism. (We assume \( \Delta \phi_A = \Delta \phi_B \) in all simulations presented here.) The transfer function is then formed using Equation 1. We currently incorporate the known filter transmission curve for the F583W filter and the quantum efficiency curve for the FGS photomultipliers that appear in the FGS Instrument Handbook (Nelan et al. 2004). The transmission and reflection coefficients of the HST telescope optics as well as other optical elements in the beam are assumed independent of wavelength over the range of interest here, defined by the F583W filter pass band to be approximately 420 to 720 nm. In Figure 1, we illustrate the color dependence of the single-star transfer function using both simulated data created with the above equations and real FGS observations. The central portions of the transfer function for three different spectral types are shown in each case, spanning a substantial range in \( B - V \) color. Although it is a small effect, the redder the star, the broader the transfer function is in both data sets, as expected from the form of Equation 2. In addition, the increase in width is comparable for the real and simulated cases, giving some confidence that the simulated data have relevant information for the analysis of the real FGS data.

Simulated binary star data can be obtained using two input spectra and providing the desired separation of the two sources along each FGS coordinate axis and the intensity ratio, specified at a particular wavelength. (For our simulations, the intensity ratio is fixed at 555.6 nm.) We show in Figure 2 typical transfer functions obtained from the simulation program for a point source and a binary star. From these scans, we generate FITS files identical in format to those from the FGS instrument, consisting of many scans across the target (nominally 60 scans in one file for the simulations discussed here). Poisson noise and dark current are added to the data to produce
statistics similar to real FGS data for known source brightness. Sections of simulated data files are shown in Figure 3. As with standard FGS analysis techniques, these data are then separated into individual scans. At this point, the standard analysis would involve cross-correlation and co-addition of the individual scans. For the Fourier method used here, we compute the spatial frequency power spectrum of each scan and add these to obtain a final power spectrum.

The binary created for Figures 2 and 3 has a substantial color difference. The primary star has spectral type G5V, whereas the secondary has spectral type M5V, thus the $B - V$ color difference is approximately 1.0. If we form the spatial frequency power spectrum of the functions in Figure 3, we obtain the functions in Figures 4(a) and 4(b). Note that the power spectrum of a point source is zero at zero spatial frequency (since the integral of the S-curve is near zero) and most of the power is contained within $\pm 22$ cycles/arcsec. For two stars with identical spectra, the transfer function would simply be the convolution of the single star transfer function with two delta functions, each of which represents one of the stars in the system. In the Fourier domain, a convolution becomes a product, and therefore one expects to obtain the product of the Fourier transform of two delta functions (i.e. a cosine-squared fringe pattern) and the single star transfer function. One can then obtain the cosine-squared function by dividing the binary power spectrum by that of a point source. Such a division is valid in the range of spatial frequencies where the data have adequate signal-to-noise ratio. As noise begins to dominate at higher spatial frequencies (absolute values greater than approximately 22 cycles/arcsec in Figure 4), the cosine-squared function will no longer be obtained. The binary power spectrum in Figure 4(b) contains fringes on top of the power spectrum shape of the point source, as expected. However, when performing the division in order to obtain the cosine-squared function, the binary fringe pattern clearly exhibits a decrease in the amplitude of the fringes at higher spatial frequencies as shown in Figure 4(c) (those in the range $\pm 15$ to $\pm 22$ cycles/arcsec). This is the signature of the color difference of the two point sources.

In the image plane, the effect of color shows up as a difference in the width of the transfer function, which arises because the interference fringes produced by the FGS are wavelength dependent and the effective wavelength depends on stellar color. As shown in the FGS Instrument Handbook (and evident from Figure 1), the main effect is simply a stretching the S-curve for redder stars. In the Fourier domain, this corresponds to a shrinking of the transform of the S-curve so that there is a decrease in signal at higher spatial frequencies. Therefore, if a binary with a substantial difference in color is observed, there will be no interference fringes in the high spatial frequencies of the divided power spectrum because the secondary has little or no signal there. So, we expect the fringes to be suppressed at higher frequencies by an envelope function, just as we see in Figure 4(c). The envelope of this decrease in amplitude is found to be closely related to a Butterworth function of order 4, $\beta(u)$, where the width $u_0$ is related to the color difference of the sources:

$$\beta(u) = \frac{1}{1 + \left(\frac{u}{u_0}\right)^8}. \quad (5)$$

The upper and lower envelopes, $E_{\pm}(u)$, of the amplitude decrease are then given by
$E_{\pm}(u) = (1 \pm \beta(u) \cdot \frac{B}{A})^2,$

(6)

where $A$ and $B$ are the intensities of the primary and secondary stars, respectively. (In Figure 4, $A$ is normalized to 1.) In Figure 4(c), a width of $u_0 = 19.1$ cycles/arcsec is shown, which is a reasonable fit to the color difference in that case. The parameter $u_0$ is expected to decrease with increasing color difference. It is convenient to parameterize the Butterworth function as follows for FGS data:

$$\beta(c, u) = \frac{1}{1 + \left(\frac{cu}{35.7 \text{ cycles/arcsec}}\right)^8},$$

(7)

The parameter $c = 35.7/u_0$ will increase with increasing color difference, and will be referred to hereafter as the color index parameter.

This effect suggests the following approach to the analysis of FGS TRANS data. One may fit in the Fourier domain not only for the separation and intensity ratio of the two components of a binary star system, but also the width of the Butterworth function that describes the attenuation of the fringe amplitude at higher frequencies. Using model results, one can convert this width into a $\Delta(B - V)$ color estimate between the primary and secondary.

### 2.2. The Fitting Program and Model Results

A program was written to perform a weighted least-squares fit of FGS spatial frequency power spectra. The weighting scheme is the same approach used by Horch et al. (2002) for speckle data analysis, namely that the weights are chosen to approximate the signal-to-noise ratio (SNR) at each location in the Fourier domain. In the case of speckle data, there is an analytical form for the SNR as a function of spatial frequency in the power spectrum. For FGS data no such function is readily available, and so we have estimated this using the data themselves. The data file is broken into five sections, and the power spectrum of each is computed. The average signal at each spatial frequency is derived, and the standard deviation of the five values for a particular spatial frequency serves as the noise value. The SNR function is then created and boxcar smoothed to obtain the weighting function for a particular FGS observation. An example of the weights of model data obtained is shown in Figure 4(c).

The data to be fitted are divided by the power spectrum of a point source, weighted according to the above, and then trial fit functions are generated that are of the following form:

$$P(u) = v_r^2(u) + v_i^2(u),$$

(8)
where $u$ is the Fourier conjugate variable to the spatial coordinate $x$, $P(u)$ is the trial power at spatial frequency $u$, and $v_r$ and $v_i$ are given by

$$v_r(u) = \beta(c, u) \cdot B \cos(2\pi x_{AB} u) + A$$

and

$$v_i(u) = \beta(c, u) \cdot B \sin(2\pi x_{AB} u),$$

where $\beta(c, u)$ is the parameterized Butterworth function previously defined, $A$ and $B$ are the intensities of the primary and secondary components, respectively, integrated through the FGS pass band (and therefore stars of different color will have a different effective wavelength due to the large width of the pass band), and $x_{AB}$ is the separation of the components along the instrument’s $x$-axis.

The power spectrum analysis will not give the sign of the separation, and so to remove this ambiguity, we calculate a portion of the bispectrum for the observation, as is done in speckle imaging with speckle data frames. (See e.g. Lohmann, Weigelt, and Wirnitzer, 1983, for more information on the bispectrum.) Briefly, for a set of transfer functions $T(x)$ with Fourier transforms $\hat{T}(u)$, the bispectrum $B(u_1, u_2)$ is defined

$$B_T(u_1, u_2) = \langle \hat{T}(u_1) \hat{T}(u_2) \hat{T}^*(u_1 + u_2) \rangle,$$

where $\langle ... \rangle$ denotes the average over all scans in the set. Using the convolution theorem once again, this may be written

$$B_T(u_1, u_2) = \hat{O}(u_1) \hat{O}(u_2) \hat{O}^*(u_1 + u_2) < \hat{S}(u_1) \hat{S}(u_2) \hat{S}^*(u_1 + u_2) >,$$

where $\hat{O}(u)$ is the Fourier transform of the true object intensity distribution and $\hat{S}(u)$ is the Fourier transform of the transfer function of a point source. Note that the terms being averaged on the right hand side are simply the bispectrum of a point source, which we may write $B_S(u_1, u_2)$. Considering only the phase of the above equation (which will be denoted “arg”) and letting $u_1 = u$ and $u_2 = \Delta u$, with $\Delta u$ a small increment in the Fourier domain, one obtains

$$\arg[B_T(u, \Delta u)] - \arg[B_S(u, \Delta u)] = \phi_O(u) + \phi_O(\Delta u) - \phi_O(u + \Delta u),$$

with $\phi_O(u)$ denoting the true phase of the object’s Fourier transform. The above is a finite difference equation that allows one to calculate $\phi_O(u + \Delta u)$ from $\phi_O(u)$ and observed bispectra of the object of interest $T$ and a point source $S$. In speckle imaging, it can be shown that $\arg[B_S(u, \Delta u)]$ has
value zero on average, but because the FGS transfer function of a point source is in theory an odd function, this same result is not so for FGS data, meaning that $\arg[B_S(u, \Delta u)]$ must be calculated and subtracted from the phase of the binary bispectrum in order to obtain the phase of $O(u + \Delta u)$. The phase at the origin is assumed zero because the object intensity is a real function. As one builds up the phase from $\phi(0) = 0$, the presence of $\phi_O(\Delta u)$ adds a linear phase term onto the result for the function $\phi_O(u)$. On the image plane, this merely translates image features, but does not affect their relative position or intensity. Therefore, one may choose this constant arbitrarily, and we set it to zero for all calculations here.

From the bispectrum an estimate of the phase of the object’s Fourier transform can therefore be determined, and when combined with the modulus of the power spectrum and inverse transformed, a reconstructed image is obtained. We then read off the placement of a component relative to the position of the primary in this function. When the separation is small, the peaks in the reconstructed image are blended, but asymmetries in the shape of the function still allow for a determination of the sign of the separation. The four fitted parameters for a one-dimensional scan are therefore $c$, $A$, $B$, and $x_{AB}$. For two-dimensional data fit simultaneously, the fifth parameter is the $y$-separation of the components, $y_{AB}$. The simplex method of Nelder and Meade (1965) is then used to find the parameters which minimize the reduced-$\chi^2$. The routine is capable of fitting either $x$- or $y$-scans independently, but for the analysis presented here, we require separations on both axes to be fit simultaneously with the same $A$, $B$, and $c$ parameters. The code is also capable of fitting a trinary system, and can easily be generalized to fit as many components as desired. The FGS transfer function amplitudes for very faint stars are known to decrease over the theoretical expectation due to the presence of dark current in the photomultipliers, but since the stars under consideration in the current work are brighter than the magnitude at which this effect becomes significant, we have made no correction for this in the fitting procedure.

Several families of FGS simulations have been created and fitted with the program discussed above. We find that there is a clear correlation between the $B - V$ color difference of the input primary and secondary, $\Delta(B - V)$, and the square of the color index parameter, $c^2$, obtained from the width of the Butterworth envelope function. In Figure 5, we show this relationship for a binary star with primary star spectral types of both F5V and G5V, a variety of secondary spectral types, and two magnitude differences. (For a given simulation family, the magnitude difference between the two stars is held fixed at a particular wavelength, in all cases here, 555.6 nm.) The noise level added to these data was that appropriate for an FGS observation of an 8th magnitude source. The curves show that the correlation has two regimes, $\Delta(B - V) < 0.20$ and $\Delta(B - V) > 0.20$. In both cases the correlation between $\Delta(B - V)$ and $c^2$ is nearly linear, but the regions have significantly different slopes. This is typical of all the simulations we have completed.

The simulation families in Figure 5 were completed in preparation for studying real FGS data of the HD 157948 system. (A more detailed study of for a wider range of primary spectral types and magnitudes is currently underway.) HD 157948 has a total $B - V$ color in the literature of 0.76, and the spectral type is listed as G5 in the Hipparcos Catalogue (ESA, 1997). The system
magnitude and $B - V$ color place it near the main sequence in the H-R diagram, indicating that the components are most likely dwarfs. We have included results for eight simulation families in Figure 5, four with an F5 primary star, and four with a G5 primary star. Of the latter, there are two that approximately describe the magnitude difference of the AB subsystem discussed in Section 3, and two that describe the AC subsystem. In each case, one simulation family has real stellar spectra as input (from the catalogue of Pickles 1998) and the other family has blackbody spectra as input. Figure 5(a) shows the results of the correlation for a primary spectral type of F5V, magnitude difference of 0.7, and separation of 35 mas, and Figure 5(b) shows the results of the correlation for a primary spectral type of F5V, magnitude difference of 2.0, and separation of 255 mas. Figures 5(c) and 5(d) are similar to Figures 5(a) and 5(b) in separation and magnitude difference respectively, but are calculated for a primary spectral type of G5V so that Figure 5(c) is similar to the AC subsystem and Figure 5(d) is similar to the AB subsystem. The change in the curve between real input spectra and blackbody spectra is at most minor, and is neglected in the linear fits here. We expect that, to first order, these curves are therefore independent of metal abundance.

Inverting these curves to give $\Delta(B - V)$ as a function of $c^2$, we obtain the results shown in Table 1. The values shown represent the best fit including points generated from both the Pickles spectra and blackbody spectra, under the assumption of no dependence on metal abundance. The columns give (1) the input spectral type of the primary star; (2) the magnitude difference, held fixed for all secondary colors; (3) the separation of the primary and secondary; (4) the range of the color index parameter appropriate for the linear coefficients in the following two columns; (5) the slope of the linear fit to the data in the $c^2$ range indicated; and (6) the $y$-intercept of the linear fit to the data for the $c^2$ range indicated. The predicted color difference based on $c^2$ is then obtained using the following formula:

$$\Delta(B - V) = ac^2 + b.$$  \hspace{1cm} (14)

The finite signal-to-noise ratio in the power spectrum of FGS data implies that the color index parameter can only be measured down to a certain value. At that point, the decrease in fringe amplitude becomes too small to be measured reliably in the presence of noise. It is also clear from the form of Equation 6 that this limit will also depend on the intensity ratio of the two sources in the system. For color differences less than $\sim 1.2$ magnitudes, the spatial frequencies that will give the most information regarding the color index parameter are between 15 and 22 cycles/arcsecond. There are 11 independent samples in the power spectrum of our FGS data between these limits. If the average signal-to-noise ratio in this range is $(S/N)_{\text{avg}}$, then we may roughly estimate the $\Delta(B - V)$ below which a value cannot be reliably determined in the following way. Assuming Gaussian errors, the effective signal-to-noise ratio of a set of $N_{\text{obs}}$ FGS observations of the target is given approximately by:
\[(S/N)_{\text{eff}} = (S/N)_{\text{avg}} \sqrt{11} \sqrt{2} \sqrt{N_{\text{obs}}}, \tag{15}\]

where \(\sqrt{11}\) arises because of the 11 independent samples, and \(\sqrt{2}\) arises due to the fact that the \(x\)- and \(y\)-axes of the FGS observation may be combined. For example, for the FGS observations of HD 157948 presented in Section 3, a typical value of \((S/N)_{\text{avg}}\) after deconvolution by a point source is 10.5, and with seven observations of the system, \((S/N)_{\text{eff}}\) is then 130.3. Let the requirement for “reliable measurement” of the color difference mean a 3\(\sigma\) drop in the envelope function at the midpoint of the above frequency range, namely at 18.5 cycles/arcsecond. In that case, 3\(\sigma\) would equal \(1/130.3 \cdot 3 = 0.023\) and \(E_{+}(18.5)\) must therefore fall to at least 97.7\% of its value at \(u = 0\). One can then calculate what value of the color index parameter is needed to achieve this for a given value of \(B/A\), whereupon one finds that for \(B/A = 1\) \((\Delta m = 0)\), \(c^2 \geq 1.46\), and for \(B/A = 0.16\) \((\Delta m = 2)\), \(c^2 \geq 2.05\) for reliable measurement of the color difference. Using Equation 14 and the values in Rows 7 and 8 of Table 1 for example, these correspond to color differences of \(\Delta(B - V) = 0.13\) and \(\Delta(B - V) = 0.26\) respectively.

### 2.3. Applying the Method to Test FGS Data

As discussed in the FGS Instrument Handbook, calibrations of single stars are taken periodically in TRANS mode. A range of colors is represented in the sample. In addition, we have obtained three observations of point sources to aid in the calibration of observations taken for our program. From our own data and the calibration data available, we have selected several point sources shown in Table 2 to conduct a further study of the Fourier method. The columns give (1) the object name; (2) the Hipparcos Catalogue number of the object, where one exists; (3) the observation date (UT); (4) the apparent \(V\) magnitude of the object; (5) the \(B - V\) color; (6) the spectral type; and (7) the number of scans in the observations.

Using these data, we can construct a “binary” scan of a given color difference by selecting two targets from the list, breaking these observations into individual scans, and then adding a scaled, shifted scan of the redder star to that of the bluer star to mimic a binary star scan. Once the binary scans are created, then the analysis can continue as previously described, including the fit in the Fourier domain to the Butterworth envelope function. By doing this with several trial “secondary” stars of different color, we can see if the simulations give consistent results with real FGS scans.

The first four entries in Table 2 have nearly the same color, which is \(0.5 \pm 0.1\) for all four stars. This is nearly centered on the \(B - V\) value for an F5V primary, and so results will be compared with the F5V simulations from the previous section. Each of the four stars has been used as a primary with all stars with \(B - V\) values redder than or equal to that of the primary used as the secondary. Therefore, the first star, HIP 3430, is used as the primary for all the stars on the list, Upgren 69 is used as the primary for all stars except HIP 3430, and HIP 57450 is used the primary for all stars on the list except HIP 3430 and Upgren 69. In each case, two binary power spectra
were constructed: one corresponding to a magnitude difference of 0.7 and separation 35 mas, and the other with magnitude difference 2.0 and separation 255 mas.

The results of this analysis are shown in Figure 6. In each plot, the y-axis position of each point is the average value of $c^2$ obtained for a given secondary using each of the (up to) four point sources as the primary and the point source for the deconvolution. The horizontal error bar marks the range in the color difference for the four primaries, and the vertical error bar is the standard deviation of the $c^2$ values obtained in each case. The linear relationships for the $c^2$-to-$\Delta(B-V)$ calibration determined from the simulations for comparable primary color (i.e. $a$ and $b$ values from Rows 1 through 4 of Table 1) are plotted with these data. The results serve to make two important points. First, note that the sources used in this study, namely those in Table 2, have fewer scans and indeed are much fainter than those of the simulation study. This means that we can anticipate significantly smaller signal-to-noise ratios in the real data and consequently significantly larger uncertainties associated with the points when constructing the calibration curves. This is particularly apparent in Figure 5(b), where without the simulation results there would be little hope of obtaining a reliable color difference even if one could measure $c^2$ very precisely from e.g. a very high signal-to-noise set of observations of a target. If one were left to determine the slope and intercept of the calibration function simply from the data points in the plot, the uncertainties of the coefficients would obviously be too large to make such a relation useful. Second, Figure 5(a) shows reasonably good agreement (and Figure 5[b] shows no inconsistency) between the results obtained from the simulated data and the values obtained from real FGS scans. Therefore, one can have some confidence when using the results in Table 1 to convert from a measured $c^2$ to a color difference even in the case of real data. Figure 5(a) suggests that the simulations have value in minimizing the uncertainty of derived color differences by effectively removing any random error associated with the coefficients while adding at most only a modest amount of systematic error.

3. Observations and Analysis of HD 157948

3.1. FGS Reductions

FGS TRANS mode observations of HD 157948 have been taken as a part of a larger project to study metal-poor spectroscopic binary stars. Seven observations of HD 157948 have been taken over the period 2001 to 2005, and the fitting program described in the previous section has been used to obtain preliminary astrometry and photometry of the system. Of the three unresolved sources observed for our program, HIP 3430, HIP 15797, and HIP 57450 (which were also used in the study presented the previous section and color data on these objects appears in Table 2), HIP 57450 has a $B-V$ color closest to that of HD 157948, and was used as the point source calibration object for this study. As discussed in the introduction, the system consists of an inner double-lined spectroscopic binary, which we will refer to as the A and B components. The intermediate component discovered by Heasley et al. will be referred to as the C component, and the wide companion discovered by
Couteau will be referred to as the D component.

All observations were taken using FGS1r with the F583W filter, with typical observation time of 2400 seconds. The number of scans per observation was nominally 55. Example sections of raw data scans in the $x$-direction are shown in Figure 7 for both the point source calibration object (namely HIP 57450) and HD 157948. These may be directly compared with the simulated data presented in Figure 3. We analyzed the data exactly as described for the simulated data, using exactly the same programs to obtain separation, position angle, intensity ratio, and estimated color difference between the components. Figure 8 shows the power spectra of both the point source and the 2004 September 27 of HD 157948 (i.e. the same two observations as shown in Figure 7). As in Figure 4(c), Figure 8(c) shows the result of the division. Figure 8(d) compares the data with the trinary power spectrum fit and shows the resulting residuals.

The astrometric results for the AB subsystem are shown in Table 3, where the columns indicate (1) the epoch of observation in Besselian years; (2) the position angle in degrees East of North; (3) the separation in milliseconds of arc (mas); (4) the position angle obtained by Heasley et al. (2002) using the standard FGS reduction methods, and (5) the separation obtained by Heasley et al. Taken together the results indicate that the Fourier astrometry is roughly consistent with that of Heasley et al., except for the 2002.0666 observation, where in both cases the separation is near the limit resolvable by FGS, and the position angles differ by approximately 30 degrees. In addition, the separation vector in this case is nearly aligned with the $x$-axis of the instrument, meaning that in the $y$-axis the system is unresolved, leading to a sizeable uncertainty in the true position angle.

The Fourier-derived position angle and separation measures in Table 3 are presented without uncertainties; typical separation uncertainties for a target comparable to HD 157948AB are approximately 1 mas using the standard image plane analysis (Nelan et al. 2004). An uncertainty in position angle can then be inferred based on

$$
\delta \theta = \arctan\left(\frac{\delta \rho}{\rho}\right),
$$

where $\rho$ is the measured separation, $\delta \rho = 1$ mas, and $\delta \theta$ is the uncertainty in the position angle. However, the Fourier method may not have the same uncertainties as the method of template fitting used in the standard image plane analysis. In general, it would be preferable to study this in detail with multiple observations of many binary stars to establish the Fourier uncertainties independently. This will be pursued in the future. At the present time, we can use the three observations in common between the Fourier method and the Heasley et al. (standard image plane) results in Table 3 to estimate typical uncertainties for the Fourier method. Consider first the three differences in separation. Assuming the uncertainties add in quadrature, each difference is a deviate drawn from a distribution with zero mean and standard deviation given by
\[
\sigma_\rho = \sqrt{(\delta \rho_s)^2 + (\delta \rho_f)^2},
\]

(17)

where \(\delta \rho_s\) is the uncertainty of the standard analysis and \(\delta \rho_f\) is the uncertainty of the Fourier analysis. In the case of the three measures in common between the two techniques, we calculate \(\sigma_\rho = 1.2\) from the table values. On the other hand, we can use the three position angles in each case together with Equation 16 to generate estimates of the linear measurement error orthogonal to the separation vector (which should on average be the same as \(\sigma_\rho\)) by converting the position angle difference into a linear difference in mas. For example, in the case of the 2002.2638 measure, the difference in the position angles is 9.78\(^\circ\), which at a separation of 25 mas is equivalent to a linear measurement error of 4.3 mas. Performing the same calculation for the other two epochs and computing another \(\sigma_\rho\), we obtain 3.3 mas. Averaging with the previous result, our final estimate for \(\sigma_\rho\) is 2.3 mas. Since the standard analysis has a known uncertainty of approximately 1 mas, this implies from Equation 17 that \(\delta \rho_f\) is approximately equal to 2 mas. Uncertainties in position angle for the Fourier technique can be obtained by using this figure and the separation of the measure in Equation 16.

We assume that the stars are not variable to the limit of our photometric accuracy, and therefore it is permissible to average the results of our seven observations. Average photometric results obtained from the Fourier method are shown in Table 4. The columns here indicate (1) the component pair, (2) the magnitude difference obtained in the FGS pass band (the effective wavelength of the filter/detector combination in this case is approximately 536 nm); (3) the \(c\) parameter obtained from the best Butterworth envelope; and (4) the implied \(\Delta (B - V)\) using the values shown in Table 1 and Equation 15. (In the case of the AB component, the values shown in line 7 of Table 1 were applied, and in the case of the AC component, values in line 6 were applied.)

To obtain the color differences, there is a complication over the simulation data presented in that the system of interest requires a trinary fit, including two color index parameters, \(c_{AB}\) and \(c_{AC}\). We constructed a trinary simulation of an 8th magnitude system with primary spectral type G5V, secondary spectral type G8V, and tertiary spectral type K7V. Therefore, the input color differences are \(\Delta (B - V)_{AB} = 0.08\) and \(\Delta (B - V)_{AC} = 0.66\). In ten independent trials, we found that, although the astrometry and intensity ratios of the companions were found reliably for this source, it was not possible to fit simultaneously for both color differences. However, if the value of \(c_{AB}\) was held fixed to a value implying a color difference of zero between the A and B components, \(c_{AC}\) was then reliably obtained, with a final average result of \(\Delta (B - V)_{AC} = 0.67 \pm 0.05\). It was then possible to fix the value of \(c_{AC}\) to the average of the ten trials and fit for \(c_{AB}\), whereupon the average result was \(\Delta (B - V)_{AB} = 0.05 \pm 0.07\). (These results we obtained using lines 6 and 7 of Table 1.) As these values are consistent with the input spectra, we therefore followed the same procedure with the trinary fits to HD 157948 FGS data.

These HD 157948 results indicate a small color difference between A and B, but a more substantial color difference between A and C, though the uncertainty in the implied \(\Delta (B - V)\) is
somewhat large. The uncertainty listed for these values is again the standard error in the mean.

### 3.2. Speckle Observations and Analysis

After the discovery of the C component of the system which was first reported in Heasley et al. (2002), a program of follow-up observations via speckle imaging was started. The separation reported by Heasley et al. was well within the capabilities of the RYTSI speckle camera, which is in use at the WIYN 3.5-m Telescope at Kitt Peak National Observatory. The speckle camera is described in Meyer et al. (2006), and the basic analysis techniques are described in Horch et al. (2002) and Horch, Meyer, and van Altena (2004). The system utilizes a large format CCD to capture a sequence of speckle images; the CCD method has been shown to produce photometry with uncertainties in the range of 0.1 to 0.15 magnitudes in most cases for a two-minute speckle observation.

Several observations of HD 157948 have been taken to date in a variety of filters. Unfortunately, none of these is better than average quality, and all but the three best observations were taken in very poor conditions. Only the three best observations will be included in the discussion here. Fortunately, of these three observations, three different filters are represented: one observation was taken with a filter of center wavelength 550.0 nm, one with center wavelength of 698.0 nm, and the third with center wavelength of 754.0 nm. The transmissions of all three filters have a full width at half maximum of approximately 40 nm. More information on these filters is available in the Meyer et al. reference mentioned above.

Starting with the widest component and working inward, the D component lies very close to the edge of the nominal field of view imposed in the standard speckle analysis, but can be seen in a few individual speckle frames. It is however, not within the limits stated in Horch, Meyer, and van Altena (2004) for speckle photometry analysis. In order to obtain a magnitude difference for this component in at least one filter, we have selected a few individual frames at 754 nm where the seeing is particularly good and the D component can clearly be seen, and computed a fit to the direct image using seeing-limited Gaussian fit functions. The result for the magnitude difference between the summed light of the A, B, and C components and the light of the D component is $\Delta m_{ABC,D} = 3.32 \pm 0.11$.

In all three speckle observations, the C component is obvious and the separations obtained from the standard power spectrum fitting procedure are consistent with those obtained with the FGS system. However, in examining the residuals of the fit in each case, it is clear that a curvature exists, potentially indicating the presence of a wide fringe pattern due to a very close component in the system (i.e. the B component), below the diffraction limit of the telescope. We attempted a trinary fit to the data in each case, whereupon we obtained separations for the third component at or below the diffraction limit. In all three cases, the position angle obtained was in the quadrant expected based on the (significantly higher precision) FGS observations. The separations were...
within a few milliseconds of arc of the predicted separation based on the preliminary FGS orbit discussed below in Section 3.3.

Due to atmospheric dispersion, which will affect the ground-based speckle data, it is dangerous to assume that these fits are definitive, however. Residual atmospheric dispersion will elongate speckles in a direction pointing toward the zenith, and will also cause a residual curvature in a binary fit to the speckle data power spectra that could masquerade as another star whose separation relative to the primary is below the diffraction limit. Nonetheless, we suggest that in the case of HD 157948, the residual dispersion should be quite small as the object was observed in all cases near the meridian and has declination $+38^\circ$, and was therefore always within a few degrees of the zenith at the time of observation. Also, as discussed in Horch, van Altena, and Franz (in prep), the expected signature of residual dispersion is color dependent in that speckles obtained in a blue filter will be more elongated than those obtained in a red filter. If interpreted as a companion star with a sub–diffraction-limited separation, one would deduce a magnitude difference that is smaller in the blue than the red, which is inconsistent with the two stars lying on the main sequence. Furthermore, residual dispersion can be identified as the principal cause of speckle elongation if the separation vector lies along the direction leading to the zenith.

In the case of the B component of HD 157948, we suggest that, at least in the case of our best speckle observation (at 754.0 nm), the signature of the unresolved B component is present in the data and that, at this wavelength, residual dispersion should be minimized. As the astrometry obtained is consistent with the FGS astrometry in both position angle and separation, we will assume that it is acceptable to use the magnitude difference of this observation in the photometric analysis as well. For the other two WIYN observations, the lack of a good match between the position angles obtained with those predicted from the orbit indicates that the signature of the unresolved B component is combined with the effects of residual dispersion, and we view the photometry as therefore not trustworthy. We indicate our final speckle results in Table 5, where the columns give (1) the Besselian year of the observation epoch; (2) the components; (3) the position angle in degrees from North through East; (4) the separation in mas; (5) the magnitude difference; (6) the center wavelength of the observation in nm, and (7) the filter full width at half maximum in nm. Uncertainties in the magnitude differences are assigned using Figure 5(b) in Horch, Meyer, and van Altena (2004) for the C component, but due to the extremely small separation in the case of the B component, one may expect that the uncertainty in the magnitude difference is probably larger than for a separation above the diffraction limit. For this reason, a value more appropriate to a low-quality observation from Horch, Meyer, and van Altena was assumed, namely 0.3 magnitudes. Although the speckle astrometry will not be used in the orbits computed and discussed in later sections of this paper, uncertainties for the C component may be estimated from Horch et al. (2002) as approximately 3 mas in separation and $1^\circ$ in position angle. Separation uncertainties in that work do not vary significantly with decreasing separation (even near the diffraction limit), and so for the one observation of the B component shown in Table 5, we suggest an estimate of 3-4 mas for the separation uncertainty is not unreasonable, allowing for
some modest loss of precision below the diffraction limit. For the position angle uncertainty of the B component, the results in Horch et al. (2002) are consistent with $\sigma_\theta(\rho) = 0.163/\rho$ in degrees, implying an uncertainty here for the B component of $4.5^\circ$. A slightly higher value, perhaps 5 to $6^\circ$ is again probably warranted due to the sub-diffraction-limited separation.

### 3.3. Preliminary Orbit

Radial velocities for the A and B components of HD 157948 exist and spectroscopic orbits have been calculated from these data first by Latham et al. (1992) and more recently by Goldberg et al. (2002). However, in order to obtain individual masses, relative astrometry is needed. We have the required data in Table 3, but it should be noted at the start that the Goldberg et al. result did not account for the (at that time, unknown) C component. Therefore, it is possible that the radial velocity amplitudes of the A and B components were affected, most likely pushing them to slightly lower values. On the other hand, the Hipparcos analysis of the system did not account for the internal motions of the stars in the system, and the distance measure implied may also be slightly in error. Indeed, the intermediate astrometric data show large residuals (D. Pourbaix, private communication). What follows here should therefore be viewed as a first attempt at understanding the system.

Pourbaix (1998) has studied the analysis of a combined visual-spectroscopic data set, which has resulted in his simulated annealing method. The method fits trial orbits to both radial velocity and relative astrometry data simultaneously and attempts to find a global minimum in reduced-$\chi^2$. He has graciously made his code available for this study, and we have used the radial velocities appearing in Goldberg et al. (2002) in combination with the Fourier astrometry in Table 3 to determine orbital parameters of this system. We assumed uncertainties in radial velocities of 0.64 km/s for the primary, and 1.06 km/s for the secondary, based on the discussion in the Goldberg et al. reference. For FGS data, a precision of 1 mas is generally quoted, but due to the small separations measured, it was warranted to increase this to 2 mas especially in view of the discussion in Section 3.1. Thus, uncertainties in separation were assigned this value for all FGS observations for the purposes of the $\chi^2$ fit, and uncertainties in position angle were determined from this value and the separation measured using Equation 16, where again $\rho$ is the measured separation, but now $\delta \rho = 2$ mas, and $\delta \theta$ is the inferred uncertainty in the position angle.

The final results are shown in Table 6 together with the orbital elements in Goldberg et al. where direct comparisons are possible. (Goldberg et al. elements have been converted to the same units of the Pourbaix results.) Note that, in the case of a double-lined system where astrometry is available to determine the true physical scale of the orbit, the solution includes a distance measurement that is independent of parallax measures. In our case, the parallax obtained from the Pourbaix code is $23.65 \pm 0.65$ mas, which is somewhat higher than both the Hipparcos value of $19.78 \pm 1.07$ (ESA, 1997) and the value appearing in the Yale Parallax Catalogue of $19.4 \pm 1.9$ (van Altena, Lee, and Hoffleit, 1995). The latter is based on the best ground-based parallax available,
which is from a series of plates taken by the U.S. Naval Observatory in Flagstaff, Arizona.

We also fitted a visual orbit to the astrometry data only using the downhill simplex code of MacKnight and Horch (2004). Orbital elements are also shown for this solution in Table 6 (referred to in the last column as MH there). This fit has a period that is consistent with the orbit of Goldberg et al., given the uncertainties in both. The Pourbaix code has a smaller period, that, if correct would imply that the Goldberg et al. period is approximately 3 sigma from the true value (using the Goldberg et al. uncertainty). This is cause for some concern, but there is a substantial difference in time between the radial velocities in the Goldberg orbit, which are from the period 1982 to 1987, and the FGS astrometry, which is from 2001 to 2005, and it may be that this is one factor complicating the combined analysis. If one takes the MH-code orbit at face value and computes the distance to the system by equating the product $a \sin i$ from the orbital parameters with the value in Goldberg et al., which is $204.8 \pm 2.0 \text{ Gm} = 1.369 \pm 0.013 \text{ AU}$, then a parallax of $21.72 \pm 0.27 \text{ mas}$ is implied.

The visual orbit obtained from the Pourbaix method is shown together with the FGS Fourier astrometry in Figure 9(a), and the MH visual orbit is shown in Figure 9(b). It is clear from these plots, as well as from Table 6, that the two orbits also give different values for the semi-major axis, inclination, and eccentricity. Considering the Pourbaix orbit, we find that the point at 2002.0666 has a large residual, and indeed the orbit prediction is that the component would be unresolvable by FGS at that point in the orbit, since the predicted separation is less than 10 mas. We have run both codes without that point, and in the case of the Pourbaix code, the orbital parameters are essentially unchanged. This is due to the fact that the radial velocities place strong constraints on the angular orbital parameters and the 2002.0666 point does not influence the semi-major axis determination very much. On the other hand, the MH code does give a different result without this observation, nearly matching the semi-major axis of the Pourbaix code. However, the absence of the point leaves only points near the maximum separation values, and does little to constrain inclination and eccentricity.

The standard deviation of the residuals in separation for the Pourbaix-code orbit is approximately 5 mas, while the same result for position angle is 10 degrees. Translating the position angle value into a linear residual perpendicular to the separation vector by multiplying by the separation, we obtain approximately 3 mas. Without the point at 2002.0666, these numbers are 4 mas and 6 degrees, respectively. Again translating the position angle number into into a linear residual, the result is 3 mas. For the MH-code orbit, we obtain 2 mas for the standard deviation of separation and 13 degrees (4.4 mas) in the position angle with all points included, and 1 mas and 8 degrees (3.9 mas) in the case where the 2002.0666 point is removed. These values are generally somewhat larger than those typically quoted for the astrometric precision of FGS1r and larger than the separation uncertainty estimate in Section 3.1, but they are not unreasonable given the small scale of the orbit, near the limit of the capabilities of the instrument.

At the present time, we consider both orbits as simply demonstrations of the Fourier method,
not final or definitive orbits of the system. The FGS data are still being analyzed using the standard FGS template fitting technique, and we anticipate that the final orbit will at a minimum include this work, since the astrometric precision is much better understood for the standard analysis. Also, Prato and her collaborators have obtained more recent spectroscopic data of the AB pair, and these results are still being analyzed. Obviously, it would be better to include new astrometric data with a more uniform coverage in orbital phase to better determine the orbital parameters. However, on balance, we suggest that the results from the Pourbaix code are an acceptable preliminary orbit since it incorporates all of the data available at this time.

3.4. H-R Diagram and Discussion

The photometric data presented, together with other data in the literature, permit the estimation of absolute $V$ magnitudes and $B-V$ color differences for all four components of the HD 157948 system. The literature values used are $B-V = 0.76$ for the system as a whole, $V$ magnitude of 8.08, which is an average of the results in the Hipparcos Catalogue and the photoelectric photometry of Carney and Latham (1987), and $\Delta V_{ABC,D} = 3.97 \pm 0.18$ from the Hipparcos Catalogue. Using the distance obtained from our orbit calculation using the Pourbaix code, the absolute $V$ magnitude is $M_V = 4.95 \pm 0.07$. We will assume no reddening for the following discussion.

The photometric information obtained from the FGS results and the speckle results is differential, and in instrumental magnitude systems. To place points on the H-R diagram, we require first absolute instrumental magnitudes of the components and then some means of conversion to, e.g., the Johnson system. To obtain these, we will again turn to the empirical spectral library of Pickles (1998) and our own library of blackbody curves that were used in the FGS simulation studies.

Absolute instrumental magnitudes are obtained by starting with a single star spectrum from the library which, when multiplied by the transmission curve of the Johnson $B$ and $V$ filters, produces a $B-V$ color of 0.76 and an absolute $V$ magnitude of 4.95 (that is, the zero point is chosen to match this value for $V$). This same spectrum is then multiplied by each of the filter transmission curves and the appropriate detector sensitivity function in turn to calculate synthetic absolute instrumental magnitudes of the star, which serve as first estimates of those of HD 157948. Of course, since HD 157948 is the combination of four stellar spectra, the result is not exact, but we will iterate to a final solution as described below.

Once estimates of the instrumental absolute magnitudes of the system are in hand, the measured instrumental magnitude differences between components can be used with these values to obtain absolute instrumental magnitudes of each component in the system. (Uncertainties in these values are propagated from the uncertainties in the magnitude differences, and we assume $\pm 0.03$ for the uncertainties in the instrumental system magnitudes.) If instrumental magnitudes exist in two or more filters, then the library of stellar spectra can be searched for the spectral type that, when synthetic magnitudes are formed using the filter transmission and detector sensitivity curves,
best matches the instrumental magnitudes for each component. We perform a $\chi^2$ minimization on the instrumental magnitudes, where a final spectral type is obtained by fitting the region near the minimum of the reduced-$\chi^2$ with a parabolic function and determining the location of the minimum. Each trial spectrum is multiplied by a factor which yields an average zero point of zero among the instrumental filters relative to the input data. In general, the minimum in reduced-$\chi^2$ will fall between two neighboring spectra in the library, but the known Johnson transmission curves can be used in combination with these two spectra to determine $M_V$ and $B - V$ values in each case. The final results for a component are obtained by linearly interpolating the $M_V$ and $B - V$ values of the two spectra bracketing the minimum in reduced-$\chi^2$ to the position in between where the minimum occurs. The component values are then combined to give a system $M_V$ and $B - V$, and if not in agreement with the values mentioned above, the method is iterated using now the four spectral types as a starting point to generate new absolute instrumental magnitudes in combination with our observed magnitude differences. The method usually converges very quickly, within < 5 iterations for the data presented here.

For the A and B components of the system, we have a $\Delta m_{\text{FGS}}$ (Table 4, effective wavelength $\sim$ 536 nm) and a $\Delta m_{754}$ from the speckle observation (Table 5). For the AC subsystem, we have three speckle magnitude differences ($\Delta m_{550}$, $\Delta m_{754}$, $\Delta m_{754}$, from Table 5) and the FGS instrumental magnitude difference $\Delta m_{\text{FGS}}$ (effective wavelength $\sim$ 536 nm), from Table 4. For the ABC-D subsystem, we have a $\Delta V$ from the Hipparcos Catalogue, and a $\Delta m_{754}$ obtained from the speckle observations. Given these data, the sensible first step is to first decouple the D component from the A, B, and C components using the method described above, whereupon we obtain $M_V = 4.98 \pm 0.07$ and $B - V = 0.74 \pm 0.04$ for the ABC subsystem.

Carrying out the iterative method on the ABC subsystem using both real stellar spectra from the catalogue of Pickles and a library of blackbody spectra, we obtain values listed in Table 7. The columns give (1) the spectral library used; (2) the component; (3) the absolute $V$ magnitude; (4) the absolute $B$ magnitude; (5) the implied $B - V$ color; and (6) the independently derived color from the Fourier technique. Since the stars in the main library of Pickles have Population I metallicities and the blackbody library spectra contain no spectral lines at all, the differences obtained are some indication of the variability of the method over metallicity. The absolute magnitudes assume a distance modulus of 3.13 ± 0.07, which is the value implied from the parallax of the preliminary Pourbaix-code orbit discussed earlier, and the uncertainties given are those associated with the photometry only. To obtain values in the rightmost column of the table, the $B - V$ color of the primary is assumed from Column 5 of the table. The Fourier values are fairly consistent with the speckle/FGS magnitude-difference results, and the uncertainties are in most cases comparable. The methods are independent, so we suggest that the best results can currently be obtained by averaging the two values. We plan to refine these first results by seeking further observations of the target in the future.

Kidger and Martín-Luis (2003) have measured the $V - K$ color of the HD 157948 system to be 1.985, with small uncertainty (less than 0.02 mags). We have checked the consistency of our
final results with this value using the total $K$ response curve available from the 2MASS website \footnote{www.ipac.caltech.edu/2mass}, which includes filter, atmospheric transmission data and detector quantum efficiency as a function of wavelength, and the Pickles spectral library. Our method described above gives estimates of the spectral type of each of the four stars, and therefore the $K$-band magnitudes can be estimated for each, just as for $V$ above, and then combined to obtain an estimate of the $V - K$ color of the system. For real Pickles spectra, we obtain $(V - K)_{ABCD} = 2.16 \pm 0.25$. Blackbody spectra give a higher result, $(V - K)_{ABCD} = 2.32 \pm 0.20$, but forcing the $B - V$ color to match the observed value results in blackbody spectra that peak more toward the red, at which point the flux in the $K$ band is higher. Given the [Fe/H] values measured for the system, we find the consistency of the real spectra and the Kidger and Martín-Luiz result encouraging.

The four stars are placed on the H-R diagram in Figures 10 and 11, where the $B - V$ location is the average of the Fourier and speckle/FGS results and the error bars in the vertical axis include those in Table 7 and the figure of 0.071 magnitudes from the distance modulus added in quadrature. Using the Yale-Yonsei isochrones (Demarque et al. 2004, and references therein), we also place in Figure 10(a) and 10(b) the isochrones corresponding to [Fe/H] = -0.433 and age 1.0 Gyr and 8.0 Gyr, respectively, and in Figures 10(c) and 10(d) the isochrones corresponding to [Fe/H] = -0.681 and ages 1.0 and 8.0 Gyrs, respectively. The Yale-Yonsei isochrones give the user a choice between two color tables for making the conversion between effective temperature and $B - V$ color, the so-called Green color tables (Green, Demarque, and King 1987) and the more recent Lejeune tables (Lejeune, Cuisinier, and Buser 1998). As discussed in Yi et al. (2001), the Green color tables, although older, still appear to match observations better in some cases. In Figure 10, we use the Lejeune tables, and in Figure 11, we show the same data using the Green tables. No alpha-element enhancement is assumed; regardless of which color table is used, isochrones with alpha-element enhancement tend to shift the isochrone to the left in the diagram, which would be poorer agreement with the photometry. The [Fe/H] values for the isochrones were chosen to approximately match the previously measured values for the system, which are -0.76, by Latham et al. (1992) and -0.5, by Goldberg et al. (2002). The mass region corresponding to that implied by the orbit in the previous section is also indicated for the A and B components. The best match of the four isochrones is that of Figure (a) in both cases, where all four data points are consistent with the isochrone, and the spectroscopic components are also consistent with the mass range implied by the orbit calculation in the previous section. Thus, our analysis clearly favors the higher metal abundance of the Goldberg spectroscopy.

As one moves to older isochrones, the mass range indicated by the orbit for the A component slides upward and becomes inconsistent with the observational data. However, due to the discrepancy in the distance between our orbit calculation and the distance implied by the Hipparcos parallax, caution is warranted. The Hipparcos distance, if correct, would shift the observational points up approximately 0.39 magnitudes. Therefore, if we compare the data to the 8 Gyr isochrone, as in
Figures 10(b) and 11(b), the mass range for the A component is well above the observed location, inconsistent even with the Hipparcos parallax result. We conclude conservatively that the system is less than 8 Gyr old. Although insufficient orbital data exist to determine masses of the C and D components of the system, the photometry suggests that approximate masses are 0.6 and 0.5 solar masses for these components respectively.

4. Conclusions

A new method of analyzing binary star data taken with the *Hubble* Space Telescope FGS system has been created that allows for both astrometric and photometric results to be obtained, including an estimate of the $B-V$ color difference between components in binary and multiple star systems. The method also gives astrometry and FGS differential photometry which is consistent with previous analysis techniques.

The method has been applied to the multiple system HD 157948, which is a hierarchical quadruple consisting of a close pair that has been well-characterized as a double-lined spectroscopic binary star, a wide pair which was discovered visually by Couteau and detected by Hipparcos, and a recently discovered component of intermediate separation. In combination with other data available, the masses of the A and B components have been determined in a preliminary way, and component magnitudes and colors have been estimated. When plotted on the H-R diagram, the system appears to be most consistent with a somewhat metal-poor isochrone of age 1 Gyr, although uncertainty in the distance to the system forces a much larger age range to be considered at present, up to approximately 8 Gyr. We conclude that, within the uncertainties of the data, theory and observation are consistent for this system. This would imply masses of approximately 0.6 and 0.5 solar masses for the C and D components in the system, respectively.

We are very grateful to Dimitri Pourbaix of the Institut d’Astronomie et d’Astrophysique, Université Libre de Bruxelles, for providing his simulated annealing code used in the preliminary orbit determination presented here and discussing the work with us, and to William van Altena of Yale University and Reed Meyer of TripAdvisor, Inc. for their collaboration on the speckle results presented. We thank David Latham of Harvard University for his help in understanding two issues regarding the spectroscopy. We are also grateful to the referee, Douglas Gies of Georgia State University, for many helpful suggestions that have substantially improved the paper. This work was funded by *Hubble* Space Telescope grants GO-09034 and GO-10197. It also made use of the Washington Double Star Catalog maintained at the U.S. Naval Observatory and the SIMBAD database, operated at CDS, Strasbourg, France.
REFERENCES


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Nelder, J. A. and Meade, R. 1965, Comp. J., 7, 308


This preprint was prepared with the AAS LaTeX macros v5.0.
Table 1. Simulations Results

<table>
<thead>
<tr>
<th>Primary Sp. Type</th>
<th>∆m at 555.6 nm</th>
<th>separation ρ (mas)</th>
<th>range</th>
<th>$c^2$ slope</th>
<th>intercept $a$</th>
<th>$b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>F5V</td>
<td>2.0</td>
<td>255</td>
<td>&lt; 2.18</td>
<td>0.321 ± 0.039</td>
<td>-0.423 ± 0.020</td>
<td></td>
</tr>
<tr>
<td>F5V</td>
<td>2.0</td>
<td>255</td>
<td>&gt; 2.18</td>
<td>0.815 ± 0.003</td>
<td>-1.501 ± 0.007</td>
<td></td>
</tr>
<tr>
<td>F5V</td>
<td>0.7</td>
<td>35</td>
<td>&lt; 2.03</td>
<td>0.470 ± 0.104</td>
<td>-0.696 ± 0.068</td>
<td></td>
</tr>
<tr>
<td>F5V</td>
<td>0.7</td>
<td>35</td>
<td>&gt; 2.03</td>
<td>1.082 ± 0.017</td>
<td>-1.938 ± 0.033</td>
<td></td>
</tr>
<tr>
<td>G5V</td>
<td>2.0</td>
<td>255</td>
<td>&lt; 2.02</td>
<td>0.178 ± 0.039</td>
<td>-0.190 ± 0.012</td>
<td></td>
</tr>
<tr>
<td>G5V</td>
<td>2.0</td>
<td>255</td>
<td>&gt; 2.02</td>
<td>0.726 ± 0.002</td>
<td>-1.297 ± 0.005</td>
<td></td>
</tr>
<tr>
<td>G5V</td>
<td>0.7</td>
<td>35</td>
<td>&lt; 1.99</td>
<td>0.251 ± 0.157</td>
<td>-0.298 ± 0.048</td>
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</tr>
<tr>
<td>G5V</td>
<td>0.7</td>
<td>35</td>
<td>&gt; 1.99</td>
<td>1.011 ± 0.012</td>
<td>-1.869 ± 0.023</td>
<td></td>
</tr>
</tbody>
</table>

Table 2. Observations used in creating binary scans from FGS data

<table>
<thead>
<tr>
<th>Object</th>
<th>HIP</th>
<th>Obs. Date</th>
<th>V</th>
<th>B − V</th>
<th>Spectral No. of Scans</th>
</tr>
</thead>
<tbody>
<tr>
<td>BD+71 31</td>
<td>3430</td>
<td>2002 May 09</td>
<td>10.20</td>
<td>0.41</td>
<td>F5</td>
</tr>
<tr>
<td>Upgren 69</td>
<td>3354</td>
<td>2002 May 09</td>
<td>9.58</td>
<td>0.50</td>
<td>F5</td>
</tr>
<tr>
<td>Upgren 69</td>
<td>3354</td>
<td>2002 Nov 11</td>
<td>9.58</td>
<td>0.50</td>
<td>F5</td>
</tr>
<tr>
<td>BD+51 1696</td>
<td>57450</td>
<td>2005 May 14</td>
<td>9.91</td>
<td>0.58</td>
<td>sdG0</td>
</tr>
<tr>
<td>BD+43 699</td>
<td>15797</td>
<td>2004 Nov 24</td>
<td>8.98</td>
<td>0.98</td>
<td>K2</td>
</tr>
<tr>
<td>HD 23877</td>
<td>57935</td>
<td>2003 Mar 06</td>
<td>9.52</td>
<td>1.10</td>
<td>K2III</td>
</tr>
<tr>
<td>SAO 185689</td>
<td>...</td>
<td>2004 Mar 04</td>
<td>9.61</td>
<td>1.50</td>
<td>K5</td>
</tr>
<tr>
<td>LatCol 1A</td>
<td>28355</td>
<td>2006 Dec 22</td>
<td>9.70</td>
<td>1.92</td>
<td>K5III</td>
</tr>
<tr>
<td>LatCol 1A</td>
<td>28355</td>
<td>2003 Dec 21</td>
<td>9.70</td>
<td>1.92</td>
<td>K5III</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Spectral Typea</th>
</tr>
</thead>
<tbody>
<tr>
<td>bThese were combined into a single file prior to processing.</td>
</tr>
</tbody>
</table>

Table 3. FGS Fourier Astrometry, HD 157948 AB

<table>
<thead>
<tr>
<th>Obs. Date (Bess. Yr.)</th>
<th>θ (°)</th>
<th>ρ (mas)</th>
<th>Heasley et al. θ (°)</th>
<th>Heasley et al. ρ (mas)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001.7390</td>
<td>230.80</td>
<td>34.4</td>
<td>230.74</td>
<td>34.2</td>
</tr>
<tr>
<td>2002.6666</td>
<td>134.50</td>
<td>12.1</td>
<td>105.83</td>
<td>12.1</td>
</tr>
<tr>
<td>2002.2638</td>
<td>51.03</td>
<td>26.0</td>
<td>41.25</td>
<td>23.9</td>
</tr>
<tr>
<td>2002.9758</td>
<td>230.25</td>
<td>34.0</td>
<td>...a</td>
<td>...a</td>
</tr>
<tr>
<td>2003.8967</td>
<td>231.13</td>
<td>25.3</td>
<td>...a</td>
<td>...a</td>
</tr>
<tr>
<td>2004.7397</td>
<td>42.87</td>
<td>24.6</td>
<td>...a</td>
<td>...a</td>
</tr>
<tr>
<td>2005.4476</td>
<td>239.33</td>
<td>33.1</td>
<td>...a</td>
<td>...a</td>
</tr>
</tbody>
</table>

| aThese observations were taken after Heasley et al. (2002) was published. |
Table 4. FGS Photometry, HD 157948 AB and AC

<table>
<thead>
<tr>
<th>Components</th>
<th>$\Delta m_{fgs}$</th>
<th>$c$</th>
<th>$\Delta(B-V)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>0.72 ± 0.14</td>
<td>1.12 ± 0.14</td>
<td>0.02 ± 0.22</td>
</tr>
<tr>
<td>AC</td>
<td>2.49 ± 0.14</td>
<td>1.55 ± 0.09</td>
<td>0.45 ± 0.20</td>
</tr>
</tbody>
</table>

*This value falls below the limit of reliable measurement as discussed in Section 2.2. However, as the interval of uncertainty easily contains this range, we believe the above is a conservative estimate for the AB color difference from the data available at this time.

Table 5. Speckle Results, HD 157948

<table>
<thead>
<tr>
<th>Obs. Date (Bess. Yr.)</th>
<th>Components</th>
<th>$\theta$</th>
<th>$\rho$</th>
<th>$\Delta m$</th>
<th>$\lambda_0$</th>
<th>$\Delta \lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2002.3228 AC</td>
<td>244.98</td>
<td>288.7</td>
<td>2.41 ± 0.20</td>
<td>550</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>2002.3228 AC</td>
<td>243.64</td>
<td>299.6</td>
<td>1.69 ± 0.10</td>
<td>698</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>2002.7901 AC</td>
<td>239.40</td>
<td>302.1</td>
<td>1.56 ± 0.10</td>
<td>754</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>2002.7901 AB</td>
<td>224.98</td>
<td>36.3</td>
<td>0.30 ± 0.30</td>
<td>754</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>2002.7901 ABC-D</td>
<td>—</td>
<td>—</td>
<td>3.32 ± 0.11</td>
<td>754</td>
<td>40</td>
<td></td>
</tr>
</tbody>
</table>

Table 6. Preliminary Orbits and Masses, HD 157948 AB

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$ (yr)</td>
<td>1.2283 ± 0.0010</td>
<td>1.22521 ± 0.00033</td>
<td>1.2279 ± 0.0023</td>
</tr>
<tr>
<td>$a$ (mas)</td>
<td>...</td>
<td>32.17 ± 0.87</td>
<td>30.20 ± 0.24</td>
</tr>
<tr>
<td>$i$ (deg)</td>
<td>...</td>
<td>94.2 ± 2.0</td>
<td>100.16 ± 0.63</td>
</tr>
<tr>
<td>$\Omega$ (deg)</td>
<td>...</td>
<td>51.5 ± 1.6</td>
<td>53.14 ± 0.15</td>
</tr>
<tr>
<td>$T$ (Besselian year)</td>
<td>1986.373 ± 0.009</td>
<td>1986.380 ± 0.010</td>
<td>1986.312 ± 0.058</td>
</tr>
<tr>
<td>$e$</td>
<td>0.1634 ± 0.0077</td>
<td>0.146 ± 0.007</td>
<td>0.2164 ± 0.0042</td>
</tr>
<tr>
<td>$\omega$ (deg)</td>
<td>356.6 ± 2.7</td>
<td>359.4 ± 3.0</td>
<td>0.6 ± 7.8</td>
</tr>
<tr>
<td>$V_0$ (km/s)</td>
<td>+3.598 ± 0.084</td>
<td>+3.516 ± 0.083</td>
<td>...</td>
</tr>
<tr>
<td>$\pi$ (mas)</td>
<td>...</td>
<td>23.65 ± 0.69</td>
<td>...</td>
</tr>
<tr>
<td>$a$ (A.U.)</td>
<td>...</td>
<td>1.3600 ± 0.0021</td>
<td>...</td>
</tr>
<tr>
<td>mass sum (sol.mas.)</td>
<td>...</td>
<td>1.678 ± 0.039</td>
<td>...</td>
</tr>
<tr>
<td>mass of $A$ (sol.mas.)</td>
<td>...</td>
<td>0.887 ± 0.030</td>
<td>...</td>
</tr>
<tr>
<td>mass of $B$ (sol.mas.)</td>
<td>...</td>
<td>0.788 ± 0.021</td>
<td>...</td>
</tr>
<tr>
<td>$K_1$ (km/s)</td>
<td>15.77 ± 0.18</td>
<td>15.68 ± 0.16</td>
<td>...</td>
</tr>
<tr>
<td>$K_2$ (km/s)</td>
<td>17.88 ± 0.27</td>
<td>17.65 ± 0.25</td>
<td>...</td>
</tr>
</tbody>
</table>
Table 7. Derived Johnson Photometry, HD 157948

<table>
<thead>
<tr>
<th>Spectral Library</th>
<th>Component</th>
<th>$M_V$ (mag)</th>
<th>$M_B$ (mag)</th>
<th>$B - V$ (mag)</th>
<th>Fourier $B - V$ (mag)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blackbody</td>
<td>A</td>
<td>5.51 ± 0.03</td>
<td>6.09 ± 0.14</td>
<td>0.58 ± 0.15</td>
<td>0.58 ± 0.15 (assumed)</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>6.21 ± 0.02</td>
<td>7.16 ± 0.10</td>
<td>0.95 ± 0.11</td>
<td>0.60 ± 0.26</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>7.92 ± 0.06</td>
<td>9.25 ± 0.12</td>
<td>1.33 ± 0.13</td>
<td>1.03 ± 0.25</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>8.96 ± 0.14</td>
<td>10.25 ± 0.14</td>
<td>1.29 ± 0.20</td>
<td>—</td>
</tr>
<tr>
<td>Pickles</td>
<td>A</td>
<td>5.51 ± 0.02</td>
<td>6.10 ± 0.14</td>
<td>0.59 ± 0.07</td>
<td>0.59 ± 0.07 (assumed)</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>6.22 ± 0.05</td>
<td>7.12 ± 0.06</td>
<td>0.90 ± 0.08</td>
<td>0.61 ± 0.23</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>7.94 ± 0.08</td>
<td>9.24 ± 0.10</td>
<td>1.30 ± 0.13</td>
<td>1.04 ± 0.21</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>9.00 ± 0.15</td>
<td>10.33 ± 0.15</td>
<td>1.33 ± 0.22</td>
<td>—</td>
</tr>
</tbody>
</table>
Fig. 1.— (a) Simulated transfer functions created as described in the text. The white-light fringes die away quickly as one moves away from the position corresponding to that of the source, giving rise to the standard S-shaped transfer function $T(x)$. The solid line is the average transfer function for an F5V star ($B - V = 0.45$), the dashed line is that of a K4V star ($B - V = 1.09$), and the dotted curve is that of an M6V star ($B - V = 1.82$). Sixty scans were co-added in each case. (b) Observed average transfer functions for three single stars of different colors. The solid curve is that of HIP 3430, an F5 star with $B - V = 0.41$, the dashed curve is HIP 15797, a K2 star with $B - V = 0.98$ (55 scans were co-added in these two cases), and the dotted curve is that of HIP 28355, a K5III star with $B - V = 1.92$ (27 scans were co-added). More information on these observations is given in Table 2.
Fig. 2.— Simulated transfer functions created as described in the text. (a) A G5V single star. (b) A G5V-M5V binary star of projected separation 180 mas and magnitude difference 2.0.

Fig. 3.— Typical raw data sections produced by the FGS simulation program. In both cases, the figures show two complete scan segments including alternating scan directions, which results in four independent estimates of the FGS transfer function. (a) A G5V single star of magnitude 8.0. (b) A binary star of magnitude 8.0, projected separation 180 mas, and magnitude difference 2.0. The primary star has spectral type G5V and the secondary M5V.
Fig. 4.— Spatial frequency power spectra of the data files shown in Figure 2. (a) The G5V single star. (b) The G5V-M5V binary star. (c) The result of dividing (b) by (a). For two identical point sources, the curve would be a cosine-squared function. However, because of the color difference between the primary and secondary, a decrease in fringe amplitude is seen at higher frequencies. The dashed curve is a Butterworth envelope of width $u_0 = 19.1$ cycles/arcsec, corresponding to color index parameter $c = 1.87$. The dotted curve shows the relative weights assigned to each spatial frequency in the fitting procedure.
Fig. 5.— The correlation of the square of the color index parameter, $c^2$, described in the text, with the $B - V$ color difference between primary and secondary. The curves show two nearly linear portions, with a break at approximately $\Delta(B - V) = 0.20$. (a) A simulation family with an F5V primary, separation 255 mas, and magnitude difference held fixed at 2.0. (b) A simulation family with an F5V primary, separation 35 mas, and magnitude difference held fixed at 0.7. (c) A simulation family with a G5V primary, separation 255 mas, and magnitude difference held fixed at 2.0. (d) A simulation family with a G5V primary, separation 35 mas, and magnitude difference held fixed at 0.7. In all four plots, filled circles are data obtained with real stellar input spectra from the catalogue of Pickles (1998), and open circles are data obtained using blackbody input spectra. The dotted lines are linear fits to all data points for the two regions $\Delta(B - V) < 0.2$ and $\Delta(B - V) \geq 0.2$. 
Fig. 6.— The correlation of the square of the color index parameter, $c^2$, with the $B - V$ color difference between primary and secondary using real FGS data of single stars. Binary scans are created by adding scans of two different point sources offset by a given separation and multiplying the “secondary” scan by the desired intensity ratio of the sources. (a) separation of 255 mas and magnitude difference 2.0. (b) Separation of 35 mas and magnitude difference of 0.7. The dotted lines in each plot are the linear fits from the F5V simulations in the previous figure, which is appropriate for the stars used as primaries in this case.
Fig. 7.— Typical raw data sections for real $x$-axis FGS data. The figures show two complete scan segments including alternating scan directions in both cases, which results in four independent estimates of the FGS transfer function. (a) HIP 57450, observed on 13 May 2005. (b) HD 157948, observed on 27 September 2004. Note that the C component of HD 157948 is clearly seen in these scans at an approximate separation of 250 samples (250 mas), and that the amplitude of the main fringe is substantially smaller than that of a single star. This feature is an indication of the presence of a small separation component whose transfer function is blended with that of the primary star.
Fig. 8.— Spatial frequency power spectra for the $x$-axis of the data files shown in Figure 6. (a) HIP 57450. (b) The 27 September 2004 observation of HD 157948. (c) The result of dividing (b) by (a). Since the FGS observations include components A, B, and C of the system, there are two superimposed fringe patterns, where the tightly-spaced fringes correspond to the C component. The B component fringe pattern reaches a minimum near 20 cycles/arcsecond, indicating that a full period is approximately 40 cycles/arcsecond, meaning that the components are separated by approximately $1/40 = 0.025$ arcseconds in the $x$-direction. The fringes of the C component appear to decrease in amplitude, indicating that there is a measurable color difference between A and C. The dotted curve shows the relative weights assigned to each spatial frequency in the fitting procedure. (d) The same data as in (c), but shown here with the fit obtained from the power spectrum fitting routine (shown in the heavy dashed line). The dotted line here shows the residuals as a function of spatial frequency.
Fig. 9.— (a) Visual orbit obtained using the simulated annealing code of Pourbaix on the combined spectroscopic-visual data set of HD 157948AB. (b) Visual orbit obtained using the downhill simplex code of MacKnight and Horch on the FGS data only. In both plots, filled circles are the positions obtained from the FGS Fourier astrometry (shown in Table 3), and the open circle is the highest-quality WIYN speckle observation (which is not used in the orbit calculations). Line segments are drawn from the observed data point to the predicted position based on the orbital ephemeris.
Fig. 10.— H-R diagram of the quadruple system HD 157948, with four different isochrones using the Yale-Yonsei isochrones and Lejeune color tables discussed in the text. The thick line segments correspond to the mass range within which the A and B components must intersect to be consistent with the orbital parameters shown in Table 6. Filled circles are values obtained using the Pickles spectral library and open circles are values obtained using blackbody spectra. The points plotted with no error bars are the positions implied if the Hipparcos parallax is used instead of the value obtained from our orbit (in which case the uncertainties would be comparable to those shown). (a) $[\text{Fe/H}] = -0.433$, age = 1.0 Gyr, (b) $[\text{Fe/H}] = -0.433$, age = 8.0 Gyr, (c) $[\text{Fe/H}] = -0.681$, age = 1.0 Gyr, (d) $[\text{Fe/H}] = -0.681$, age = 8.0 Gyr.
Fig. 11.— H-R diagram of the quadruple system HD 157948, with four different isochrones using the Yale-Yonsei isochrones and Green color tables discussed in the text. The thick line segments correspond to the mass range with which the A and B components must intersect to be consistent with the orbital parameters shown in Table 6. Filled circles are values obtained using the Pickles spectral library and open circles are values obtained using blackbody spectra. The points plotted with no error bars are the positions implied if the Hipparcos parallax is used instead of the value obtained from our orbit (in which case the uncertainties would be comparable to those shown). (a) [Fe/H] = -0.433, age = 1.0 Gyr, (b) [Fe/H] = -0.433, age = 8.0 Gyr, (c) [Fe/H] = -0.681, age = 1.0 Gyr, (d) [Fe/H] = -0.681, age = 8.0 Gyr.