Astronomy 635: Fundamental of Astrophysics

Problem Set 1. – Due 9/18

1. Express the vorticity matrix in terms of the curl of the velocity field.
2. Show that in three dimensions 7 numbers are required to characterize the local relative velocity field. What are they? Identify the numbers required for a 2-dimensional velocity field?
3. Evaluate the shear, vorticity and expansion for the 2-dimensional velocity field \( \vec{v} \perp \vec{r}, |\vec{v}| \propto |\vec{r}| \). This is just rigid body rotation characteristic, for example, of the central regions of a disk galaxy.
4. Evaluate the shear, vorticity and expansion for the 2-dimensional Keplerian velocity field: \( v(r) \propto r^{-\frac{1}{2}} \) characteristic of the solar system, Saturn's ring plane and the Kuiper belt.
5. A more general class of rotation for a massive disk follows from Newton's laws:
\[
\frac{v^2}{r} = \frac{GM(r)}{r^2}
\]
\[
v(r) \propto \sqrt{\frac{M(r)}{r}}
\]
For which the Keplerian field is just a special case with \( M(r) \) a constant. Many disk models assume a power law density falloff which results in power law behavior for \( M(r) \) so assuming \( M(r) \propto r^\gamma \) find the shear, vorticity and expansion of the disk velocity field. Evaluate these coefficients for the case \( \gamma = 1, M(r) \propto r \) for which \( v \) is independent of radius. This is the well known flat rotation curve characteristic of the outer regions of disk galaxies. Can we say anything about the use of shear and vorticity fields as probes galactic structure?