Homework 4

1. The free-free absorption coefficient of a pure hydrogen plasma is given by

$$\kappa_\nu^{\mathrm{ff}} = n_e n_p \sigma_0 \nu^{-3} T^{-1/2} \left(1 - e^{-\frac{h\nu}{kt}}\right)$$

where n_e and n_p are the electron and proton number densities, respectively (given in cm^{-3}), ν is the frequency in Hz and T the temperature in K. In these units the value of σ_0 is then $\sigma_0 = 3.7 \times 10^8$.

Calculate the Rosseland mean opacity for κ_ν^{ff}. Discuss its temperature and density dependence. Compare with electron scattering:

$$\kappa_{\text{e-sc}} = n_e \sigma_e$$

where $\sigma_e = 6.6 \times 10^{-25}$ cm^2. Plot the borderline in the (log n_e, log T) plane, where $\kappa_{\text{e-sc}}$ and $\bar{\kappa}^{\mathrm{ff}}$ are equal and indicate where one is larger than the other.

(4 points)

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2. In an ionized hydrogen plasma the total absorption coefficient is a combination of Thomson scattering and true absorption. Write a small subroutine and calculate

$$\kappa_R = < \kappa_\nu^{\mathrm{ff}} + \kappa_{\text{e-sc}} >_{\mathrm{Rosseland}}$$

Test the program using the results of Exercise 1. Discuss the values of κ_R at the borderline where $\kappa_{\text{e-sc}}$ and $\bar{\kappa}^{\mathrm{ff}}$ are equal.

(3 points)

3. Calculate temperature, pressure and particle density stratifications in a stellar atmosphere with T_{eff} = 20000 K and g=10^4 cm s^{-2} and g=10^2 cm s^{-2} .

Use: $T^4 = \frac{3}{4} T_{\mathrm{eff}}^4 (\tau_R + 2/3)$ and $$\frac{dP}{d\tau_R} = g\frac{\rho}{\kappa_R}$$

as the basic equations.

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Write a subroutine for numerical solution but discuss the solution analytically first for the case $\kappa_{\text{e-sc}} >> \kappa_{\nu}^{\text{ff}}$.

Use τ_R (or its logarithm) as the depth scale.

Discuss the solutions for different gravity g.

Give electron densities at τ_R = 0.1, 1.0, 10.0.

Assume $P = nkT$ for the equation of state with $n = n_e + n_p$ (n = total particle density, n_e = electron density, n_p = proton density) and $n_e = n_p$. k is the Boltzmann constant ($k = 1.38 \times 10^{-16}$ erg deg^{-1}).

To relate the mass density ρ to particle densities use $m_p = 1.67 \times 10^{-24}$ g for the proton mass.

(6 points)

Please return homework on Monday, October 17

!!! This is a lot of work. Start early !!!!