



Homework 6

1. Hot stars have wind velocity fields of the form:
- $$v(r) = v_{\infty} \left(1 - \frac{1}{r}\right)^{\beta}$$

Calculate and plot line interaction surfaces in the full (p, z) plane for $\beta = 0.5$ and $\beta = 4.0$. Describe the numerical algorithm of your calculations and discuss the results.

5 points

2. The goal of this exercise is to develop a simple program to calculate line profiles in stellar winds for the two most important cases (pure emission lines and P-Cygni profiles), and to apply this code to the analysis of observed spectra. Refer to pages II.6.13 – II.6.36 of the lecture manuscript. We assume a velocity field of the form

$$v(r) = v_{\infty} \left(1 - \frac{1}{r}\right)^{\beta}$$

which means that you have to avoid $x = 0$ in the calculations (why?). Assume the following approximation for the radius of the interaction region at $p = 1$:

$$r_1(x) = 2 r_{min} + \frac{1}{2} \left(\frac{|x|}{v_{\infty}} - 1 \right)$$



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a) scattering line, P-Cygni profile (p. II.6.32): assume

$$S_L = \frac{I_C}{2} r^{-3}, \quad \tau_S \gg 1$$

Then the line profile is given by:

$$P(x) = \int_{r_{min}(x)}^{\infty} \left(1 + \sigma \frac{x^2}{v^2}\right) r^{-2} dr \quad x > 0$$

$$P(x) = 1 + \int_{r_1(x)}^{\infty} \left(1 + \sigma \frac{x^2}{v^2}\right) r^{-2} dr \quad x < 0$$

Write a numerical code to calculate a full profile. Test the code using the examples and the discussion given in the lectures. Download the UV CIV resonance line profile of a hot star and try to fit the profile with your calculations by varying (and thus determining) v_∞ (in km/s) and β .

When comparing your calculations with the observations note that

$$\left(\frac{x}{v_\infty}\right) \frac{v_\infty}{c} = \frac{\Delta\nu}{\nu_0} \approx -\frac{\Delta\lambda}{\lambda_0}$$



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You can improve your fit by convolving the calculated profile with a Gaussian of FWHM corresponding to 200 km/s accounting for turbulence in the wind.

b) thermal/recombination line in pure emission (p. II.6.21): assume

$$S_L = I_C \quad \tau_S \ll 1; \quad k(r) \sim \rho^2(r) \sim \frac{1}{r^4 v^2(r)}$$

Then the profile is given by:

$$P(x) = 1 + B \int_{r_1(x)}^{\infty} \frac{1}{r^2 \frac{v^3}{v_\infty^3}} dr$$

Write a numerical code to calculate a full profile. Download the H α line profile of a hot star and try to fit the profile with your calculations by varying (and thus determining) B and β . You can improve your fit by convolving the calculated profile with a Gaussian of FWHM corresponding to 100 km/s accounting for rotation. Assume the value of v_∞ as determined in a). If the value of your best fits of β differ between a) and b), do not worry. Maybe you have an explanation.

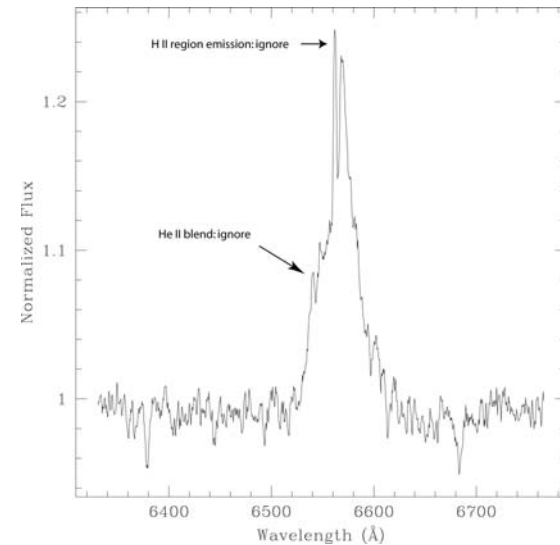
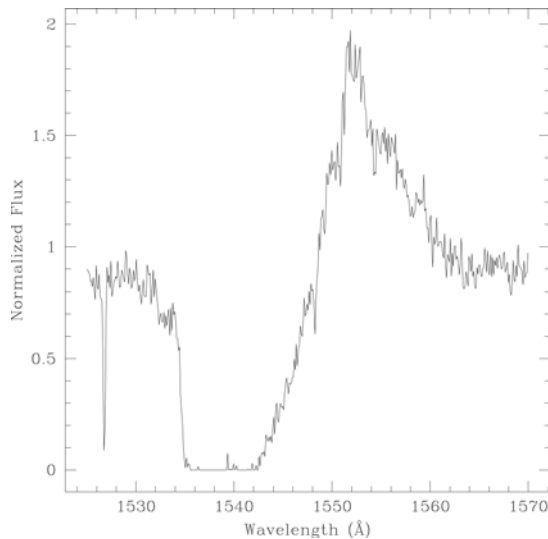
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Do not expect a perfect fit for a) or b). We make a lot of simplifying assumptions. Our goal is only to understand the basics of stellar wind spectral diagnostics.

8 points

Return Homework by Monday, November 28.

The line profiles of the UV CIV line and of the H α line can be downloaded in ASCII format from my website.



The CIV line has two components at $\lambda \approx 1548, 1551$ Å. Try either as one single line at $\lambda = 1549.5$ Å, or overplot two separate calculations for the two components. For H α concentrate mostly on fitting the red wing.