Homework 6

1. Hot stars have wind velocity fields of the form:

\[ v(r) = v_\infty \left(1 - \frac{1}{r}\right)^\beta \]

Calculate and plot line interaction surfaces in the full \((p,z)\) plane for \(\beta = 0.5\) and \(\beta = 4.0\).

Describe the numerical algorithm of your calculations and discuss the results.

5 points

2. The goal of this exercise is to develop a simple program to calculate line profiles in stellar winds for the two most important cases (pure emission lines and P-Cygni profiles), and to apply this code to the analysis of observed spectra. Refer to pages II.6.13 – II.6.36 of the lecture manuscript. We assume a velocity field of the form

\[ v(r) = v_\infty \left(1 - \frac{1}{r}\right)^\beta \]

which means that you have to avoid \(x = 0\) in the calculations (why?). Assume the following approximation for the radius of the interaction region at \(p = 1\):

\[ r_1(x) = 2 r_{\text{min}} + \frac{1}{2} \left(\frac{|x|}{v_\infty} - 1\right) \]
a) scattering line, P-Cygni profile (p. II.6.32): assume

Then the line profile is given by:

\[ S_L = \frac{I_C}{2} r^{-3}, \quad \tau_S \gg 1 \]

\[ P(x) = \int_{r_{\text{min}}(x)}^{\infty} \left( 1 + \sigma \frac{x^2}{v^2} \right) r^{-2} dr \quad x > 0 \]

\[ P(x) = 1 + \int_{r_1(x)}^{\infty} \left( 1 + \sigma \frac{x^2}{v^2} \right) r^{-2} dr \quad x < 0 \]

Write a numerical code to calculate a full profile. Test the code using the examples and the discussion given in the lectures. Download the UV CIV resonance line profile of a hot star and try to fit the profile with your calculations by varying (and thus determining) \( v_\infty \) (in km/s) and \( \beta \).

When comparing your calculations with the observations note that

\[ \left( \frac{x}{v_\infty} \right) \frac{v_\infty}{c} = \frac{\Delta \nu}{\nu_0} \approx -\frac{\Delta \lambda}{\lambda_0} \]
You can improve your fit by convolving the calculated profile with a Gaussian of FWHM corresponding to 200 km/s accounting for turbulence in the wind.

b) thermal/recombination line in pure emission (p. II.6.21): assume

\[ S_L = I_C \quad \tau_S \ll 1; \quad k(r) \sim \rho^2(r) \sim \frac{1}{r^4 v^2(r)} \]

Then the profile is given by:

\[ P(x) = 1 + B \int_{r_1(x)}^{\infty} \frac{1}{r^2} \frac{v^3}{v^3_{\infty}} \, dr \]

Write a numerical code to calculate a full profile. Download the H\(\alpha\) line profile of a hot star and try to fit the profile with your calculations by varying (and thus determining) B and \(\beta\). You can improve your fit by convolving the calculated profile with a Gaussian of FWHM corresponding to 100 km/s accounting for rotation. Assume the value of \(v_{\infty}\) as determined in a). If the value of your best fits of \(\beta\) differ between a) and b), do not worry. Maybe you have an explanation.
Homework 6

Do not expect a perfect fit for a) or b). We make a lot of simplifying assumptions. Our goal is only to understand the basics of stellar wind spectral diagnostics.

Return Homework by Monday, November 28.

The line profiles of the UV CIV line and of the Hα line can be downloaded in ASCII format from my website.

The CIV line has two components at $\lambda \approx 1548, 1551$ Å. Try either as one single line at $\lambda = 1549.5$ Å, or overplot two separate calculations for the two components. For Hα concentrate mostly on fitting the red wing.