# 8. Line transfer

The two-level atom

Milne-Eddington model

curve of growth

10/16/2003

# Two-level approximation

We consider schematic line-formation cases with easy solution

highly simplified: not accurate, but provide insight into the mechanisms at work in real stellar atmospheres.

Consider an atomic model with only two important levels: lower *l* and upper *u*.

Although highly simplified, it well approximates the situation for some lines, *e.g.* resonance lines from the ground state.

# Two-level approximation

Rate equations (statistical equilibrium)

$$n_i \sum_{j \neq i} (R_{ij} + C_{ij}) + n_i (R_{ik} + C_{ik}) = \sum_{j \neq i} n_j (R_{ji} + C_{ji}) + n_p (R_{ki} + C_{ki})$$

Consider two levels *u* and *l*: isolating the transitions between them

$$n_l(R_{lu}+C_{lu}) + n_l \sum_{j \neq l,u} (R_{lj}+C_{lj}) + n_l (R_{lk}+C_{lk}) = n_u(R_{ul}+C_{ul}) + \sum_{j \neq l,u} n_j (R_{jl}+$$

and *neglecting* all transitions involving j ≠ l,u, plus recombinations/ionizations:

$$\Rightarrow \quad n_l(R_{lu} + C_{lu}) = n_u(R_{ul} + C_{ul})$$

# Two-level approximation

substituting for the R coefficients:

$$n_l\left(B_{lu}\int_0^\infty \varphi_\nu J_\nu d\nu + C_{lu}\right) = n_u\left(A_{ul} + B_{ul}\int_0^\infty \varphi_\nu J_\nu d\nu + C_{ul}\right)$$

assuming collision rates dominate over radiative rates $\quad n_l C_{lu} = n_u C_{ul}$

remembering that $\quad C_{lu} = \left(\frac{n_u}{n_l}\right)^* C_{ul} = \frac{g_u}{g_l} e^{-E_{ul}/kT}\, C_{ul}$

$$\Longrightarrow \quad n_l \frac{g_u}{g_l} e^{-E_{ul}/kT} C_{ul} = n_u C_{ul}$$

$$\Longrightarrow \quad \frac{n_u}{n_l} = \frac{g_u}{g_l} e^{-E_{ul}/kT} = \left(\frac{n_u}{n_l}\right)^{\mathrm{LTE}}$$

LTE

# Two-level approximation

## Calculation of the line source function

$$\kappa_\nu^{\rm line} = (n_l B_{lu} - n_u B_{ul})\, \varphi_\nu \frac{h\nu}{4\pi} \qquad \epsilon_\nu^{\rm line} = n_u A_{ul}\, \varphi_\nu \frac{h\nu}{4\pi}$$

$$S_\nu^{\rm line} = \frac{\epsilon_\nu^{\rm line}}{\kappa_\nu^{\rm line}} = \frac{n_u A_{ul}}{n_l B_{lu} - n_u B_{ul}} = \frac{A_{ul}}{\frac{n_l}{n_u} B_{lu} - B_{ul}}$$

with $$B_{lu} = \frac{g_u}{g_l} B_{ul} \qquad A_{ul} = \frac{2h\nu^3}{c^2}\, B_{ul}$$

$$S_\nu^{\rm line} = \frac{2h\nu^3}{c^2} \frac{1}{\frac{n_l\, g_u}{n_u\, g_l} - 1}$$

***Note: this is the general expression for the line source function in NLTE. It is always valid (not only in 2-level approximation). What is different in the general case, is how $n_l$, $n_u$ are computed***

# Two-level approximation

LTE

If we substitute $$\frac{n_u}{n_l} = \frac{g_u}{g_l} e^{-E_{ul}/kT} = \left(\frac{n_u}{n_l}\right)^*$$ $$E = h\nu$$

we recover the Planck function $$S_\nu^{\text{line}} = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1} = B_\nu(T)$$

For the 2-level atom we found $$n_l \left(B_{lu} \int_0^\infty \varphi_\nu J_\nu d\nu + C_{lu}\right) = n_u \left(A_{ul} + B_{ul} \int_0^\infty \varphi_\nu J_\nu d\nu + C_{ul}\right)$$

$$\Rightarrow \quad \frac{n_u}{n_l} = \frac{1 + \frac{c^2}{2h\nu^3} \int \varphi_{\nu'} J_{\nu'} d\nu' + C_{ul}/A_{ul}}{\frac{g_u}{g_l} \left[\frac{c^2}{2h\nu^3} \int \varphi_{\nu'} J_{\nu'} d\nu' + e^{-h\nu/kT} C_{ul}/A_{ul}\right]}$$

and substituting in $S_\nu$:

# Two-level approximation

$$S_\nu^{\text{line}} = \frac{2h\nu^3}{c^2} \frac{\frac{c^2}{2h\nu^3}\int \varphi_{\nu'} J_{\nu'} d\nu' + e^{-h\nu/kT} C_{ul}/A_{ul}}{1 + \frac{C_{ul}}{A_{ul}}(1 - e^{-h\nu/kT})} =$$

$$= \underbrace{\frac{1}{1 + \frac{C_{ul}}{A_{ul}}(1 - e^{-h\nu/kT})}}_{1-\varepsilon} \int \varphi_{\nu'} J_{\nu'} d\nu' + \underbrace{\frac{2h\nu^3}{c^2} \frac{e^{-h\nu/kT} C_{ul}/A_{ul}}{1 + \frac{C_{ul}}{A_{ul}}(1 - e^{-h\nu/kT})}}$$

$$\underbrace{\frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}}_{B_\nu(T)} \; \underbrace{\frac{(1 - e^{-h\nu/kT}) C_{ul}/A_{ul}}{1 + \frac{C_{ul}}{A_{ul}}(1 - e^{-h\nu/kT})}}_{:= \varepsilon}$$

**1 – ε**

**$B_\nu$ (T)**

**:= ε**

# Two-level approximation

scattering term

thermal term

$$S_\nu^{\text{line}} = (1-\epsilon)\int_0^\infty \varphi_{\nu'} J_{\nu'} d\nu' + \epsilon\, B_\nu(T) = \frac{\int_0^\infty \varphi_{\nu'} J_{\nu'} d\nu' + \epsilon'\; B_\nu(T)}{1+\epsilon'}$$

$$\epsilon := \frac{\left(1-e^{-h\nu/kT}\right) C_{ul}/A_{ul}}{1+\frac{C_{ul}}{A_{ul}}\left(1-e^{-h\nu/kT}\right)} = \frac{\epsilon'}{1+\epsilon'}$$

destruction probability

photons are either destroyed into thermal pool or scattered without change in frequency and isotropically

photons are created in thermal processes ($\varepsilon\, B_\nu$)

Comparison with Chapter 3:
Continuum source function with
true absorption + scattering

$$S_\nu = \frac{\kappa_\nu}{\kappa_\nu + \sigma_\nu} B_\nu + \frac{\sigma_\nu}{\kappa_\nu + \sigma_\nu} J_\nu$$

→ line source function has similar terms except that we also allow for non-coherent scattering

# Two-level approximation

deep layers: collisions dominate → $\varepsilon' \gg 1$ or $\varepsilon = 1$ *thermal* term dominant
higher layers: collisions non-important → $\varepsilon' \approx 0$ $\varepsilon = 0$ *scattering* term dominant

a) $C_{ul} \gg A_{ul}$ $\varepsilon = 1$ → $S_\nu = B_\nu(T)$ LTE

b) $C_{ul} \ll A_{ul}$ $\varepsilon \approx 0$ → $S_\nu = \int \varphi_{\nu'} J_{\nu'} d\nu'$ extreme non-LTE

Chapter 3: $\mathbf{S_\nu = J_\nu}$ for pure *coherent* scattering
now $S_\nu = \int \varphi_{\nu'} J_{\nu'} d\nu'$ *non-coherent* scattering

# Two-level approximation

This second case ($S_\nu \neq B_\nu$) has a macroscopic interpretation in terms of *scattering*

$$S_\nu^{\text{line}} = \frac{\epsilon_\nu^{\text{line}}}{\kappa_\nu^{\text{line}}} = \int_0^\infty \varphi_{\nu'} J_{\nu'} d\nu' \quad \Longrightarrow \quad \epsilon_\nu^{\text{line}} = \kappa_\nu^{\text{line}} \int_0^\infty \varphi_{\nu'} J_{\nu'} d\nu'$$

for a narrow absorption profile function $\varphi_{\nu'} \approx \delta(\nu' - \nu)$

$$\Longrightarrow \quad \epsilon_\nu^{\text{line}} = \kappa_\nu^{\text{line}} J_\nu$$

as in a coherent scattering process (e.g. Thomson scattering)
*complete redistribution: emission and absorption profiles identical*

microscopically there is a photon absorption *l* → *u* followed by re-emission

2-level atom is a special NLTE case

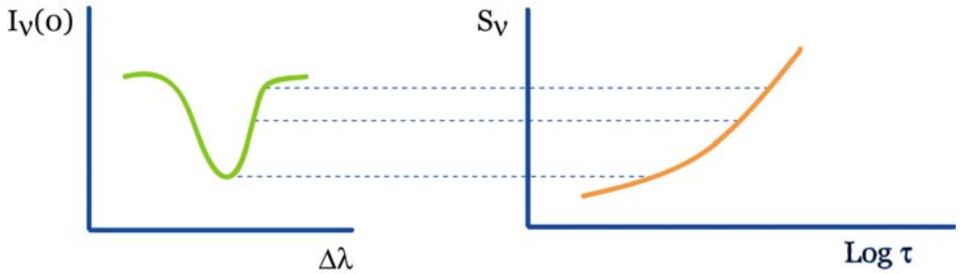
in general the coupling between $J_\nu$, $n_i$ and $S_\nu$ is far more complicated

# Two-level approximation

Moving outward in the photosphere scattering term dominates

At some point we reach the region where photons are being lost from the star (small optical depth)
→ $J_\nu$ decreases with height → $S_\nu$ decreases with height

*→ absorption line*




adapted from Gray, 92

*mapping between source function (decreasing outward) and line profile*

*line absorption coefficient larger at line center → see higher layers*

*wings formed in deeper layers than line core*

# The Milne-Eddington model

We have defined *line* and *continuum* coefficients for absorption and emission

Total absorption coefficient is $\kappa_\nu = \kappa_\nu^C + \kappa_\nu^L + \sigma$

with $\sigma$ from scattering in the continuum

The *line* optical depth is $d\tau_\nu = \left(\kappa_\nu^C + \kappa_\nu^L\right)\,dx$
(larger than in the *continuum!*)

**Qualitative line formation**

Barbier-Eddington relation: $I(0,\mu) \approx S(\tau=\mu)$

In LTE: $S_\nu = B_\nu(T)$

T decreases outward → $S_\nu$ decreases outward → absorption line

# The Milne-Eddington model

Transfer equation:

$$\mu \frac{dI_\nu(\mu)}{dx} = (\kappa_\nu^C + \kappa_\nu^L + \sigma) I_\nu - \epsilon_\nu^C - \epsilon_\nu^L - \sigma J_\nu$$

Without dealing with the general case for the computation of all coefficients we assume:

- LTE in the continuum $\frac{\epsilon_\nu^C}{\kappa_\nu^C} = B_\nu(T)$

- scattering negligible in the continuum $\sigma \ll \kappa_\nu^C$

- 2-level atom in the line

# The Milne-Eddington model

$$\mu \frac{dI_\nu}{dx} = (\kappa_\nu^C + \kappa_\nu^L) \left[ I_\nu - \frac{\kappa_\nu^C}{\kappa_\nu^C + \kappa_\nu^L} B_\nu - \frac{\kappa_\nu^L}{\kappa_\nu^C + \kappa_\nu^L} S_\nu^L \right]$$

with $\beta_\nu = \frac{\kappa_\nu^L}{\kappa_\nu^C}$ $\quad d\tau_\nu = (\kappa_\nu^C + \kappa_\nu^L) dx = \kappa_\nu^C (1 + \beta_\nu) dx$

$\lambda_\nu = \frac{1 + \epsilon\beta_\nu}{1 + \beta_\nu}$ with ε from $S_\nu^{\text{line}} = (1-\epsilon) \int_0^\infty \varphi_{\nu'} J_{\nu'} d\nu' + \epsilon B_\nu(T)$

⇒

$$\mu \frac{dI_\nu}{d\tau_\nu} = I_\nu - \lambda_\nu B_\nu - (1 - \lambda_\nu) \int_0^\infty \varphi_\nu J_\nu d\nu$$

# The Milne-Eddington model

**assumptions** *(for analytical solution)*

1. $\lambda_\nu$, $\varepsilon$ and $\beta_\nu$ constant with depth
2. $B_\nu$ linear in continuum optical depth

$$B_\nu = a + b\,\tau_c$$

$$d\tau_c = \frac{d\tau_\nu}{1+\beta_\nu} \qquad \tau_c = \frac{\tau_\nu}{1+\beta_\nu}$$

3. $\int\limits_0^\infty \varphi_\nu J_\nu d\nu \approx J_\nu$

plus the Eddington approximation $K_\nu = \frac{1}{2}\int\limits_{-1}^{1} I_\nu \mu^2 d\mu = \frac{1}{3} J_\nu$

$$H_\nu(0) = \frac{1}{\sqrt{3}}\,J_\nu(0)$$

boundary condition (without proof)

# The Milne-Eddington model

$$\mu \frac{dI_\nu}{d\tau_\nu} = I_\nu - \lambda_\nu B_\nu - (1-\lambda_\nu) J_\nu$$

$$\frac{1}{2}\int_{-1}^{1} ...d\mu : \qquad \frac{dH_\nu}{d\tau_\nu} = J_\nu - \lambda_\nu B_\nu - (1-\lambda_\nu) J_\nu$$

$$\frac{1}{2}\int_{-1}^{1} ...\mu d\mu : \qquad \frac{dK_\nu}{d\tau_\nu} = H_\nu$$

differentiate this equation again

$$\frac{d^2 K_\nu}{d\tau_\nu^2} = J_\nu - \lambda_\nu B_\nu - (1-\lambda_\nu) J_\nu$$

Eddington approximation

$$\frac{1}{3}\frac{d^2 J_\nu}{d\tau_\nu^2} = \lambda_\nu (J_\nu - B_\nu)$$

$B_\nu$ is linear in $\tau$: $$\frac{1}{3}\frac{d^2 B_\nu}{d\tau_\nu^2} = 0$$

# The Milne-Eddington model

$$\frac{1}{3}\frac{d^2}{d\tau_\nu^2}(J_\nu - B_\nu) = \lambda_\nu (J_\nu - B_\nu)$$

solution:

$$J_\nu - B_\nu = \alpha_\nu \; e^{-\sqrt{3\lambda_\nu}\;\tau_\nu} + \gamma_\nu \; e^{\sqrt{3\lambda_\nu}\;\tau_\nu}$$

boundary conditions

$$\tau_\nu \to \infty : J_\nu \to B_\nu \to \gamma_\nu = 0$$

$$\tau_\nu = 0 : H_\nu(0) = \frac{1}{\sqrt{3}} J_\nu(0)$$

we use this boundary condition to determine $\alpha_\nu$ on the next 2 pages

# The Milne-Eddington model

from $$\frac{dK_\nu}{d\tau_\nu} = \frac{1}{3}\frac{dJ_\nu}{d\tau_\nu} = H_\nu \qquad \left.\frac{1}{3}\frac{dJ_\nu}{d\tau_\nu}\right|_{\tau_\nu=0} = H_\nu(0) = \frac{1}{\sqrt{3}}J_\nu(0)$$

$$\Rightarrow J_\nu(0) = \alpha_\nu + B_\nu(0) = \alpha_\nu + a = \frac{1}{\sqrt{3}}\left.\frac{dJ_\nu}{d\tau_\nu}\right|_{\tau_\nu=0}$$

from $$J_\nu - B_\nu = \alpha_\nu\, e^{-\sqrt{3\lambda_\nu}\,\tau_\nu} + \gamma_\nu\, e^{\sqrt{3\lambda_\nu}\,\tau_\nu}$$

and $$B_\nu = a + b\,\tau_c$$

$$\frac{1}{\sqrt{3}}\left.\frac{dJ_\nu}{d\tau_\nu}\right|_{\tau_\nu=0} = \frac{1}{\sqrt{3}}\left[-\alpha_\nu\sqrt{3\lambda_\nu} + \frac{b}{1+\beta_\nu}\right] = \alpha_\nu + a$$

$$\frac{dB_\nu}{d\tau_\nu} = \frac{dB_\nu}{d\tau_c}\frac{1}{1+\beta_\nu}$$

# The Milne-Eddington model

$$\alpha_\nu = \frac{\frac{b}{1+\beta_\nu} - \sqrt{3}a}{(\sqrt{3} + \sqrt{3\lambda_\nu})}$$

define $$p_\nu = \frac{b}{1+\beta_\nu}$$

$$J_\nu(\tau) = a + p_\nu \tau_\nu + \frac{p_\nu - a\sqrt{3}}{\sqrt{3} + \sqrt{3\lambda_\nu}}\ e^{-\sqrt{3\lambda_\nu}\ \tau_\nu}$$

$$H_\nu(0) = \frac{1}{\sqrt{3}} J_\nu(0) = \frac{a}{\sqrt{3}} + \frac{p_\nu - a\sqrt{3}}{3(1+\sqrt{\lambda_\nu})} = \frac{1}{3}\,\frac{p_\nu + a\sqrt{3\lambda_\nu}}{1+\sqrt{\lambda_\nu}}$$

thermalization depth: for $\tau_\nu \geq \frac{1}{\sqrt{\lambda_\nu}}$

$J_\nu \rightarrow B_\nu$

$J_\nu < B_\nu$ in outer parts of atmosphere

# The Milne-Eddington model

**compute the line profile**

residual flux

$$R_\nu = \frac{H_\nu(0)}{H_c(0)}$$

for $H_c$:

$$\beta_\nu = \frac{\kappa_\nu^L}{\kappa_\nu^C} = 0 \Longrightarrow p_\nu = b \quad \lambda_\nu = 1$$

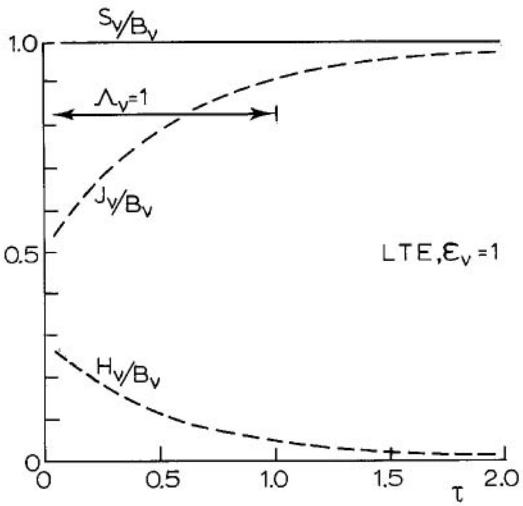
⇨

$$H_c(0) = \frac{1}{3}\,\frac{b + a\sqrt{3}}{2}$$

$$R_\nu = \frac{H_\nu(0)}{H_c(0)} = 2\,\frac{p_\nu + \sqrt{3\lambda_\nu}a}{(1+\sqrt{\lambda_\nu})(b+\sqrt{3}a)}$$

# The Milne-Eddington model

***a) case ε = 1 (LTE: pure absorption lines)***




**Zwaan, 93, Lecture notes**

*isothermal atmosphere ($B_\nu = a$)*

$$S_\nu^{\mathrm{L}} = B_\nu$$

$$S_\nu^{\mathrm{c}} = B_\nu$$

$$\lambda_\nu = \frac{1+\epsilon\beta_\nu}{1+\beta_\nu} = 1$$

$$\Rightarrow \quad R_\nu = \frac{\frac{b}{1+\beta_\nu} + \sqrt{3}a}{b+\sqrt{3}a}$$

emergent flux determined by surface value of Planck function

for strong line: $\beta_\nu = \kappa_L / \kappa_c \gg 1$

$$R_\nu = \frac{\sqrt{3}a}{b+\sqrt{3}a} = \frac{B_\nu(0)}{B_\nu(\tau_c = 1/\sqrt{3})} \neq 0$$

# The Milne-Eddington model

in LTE the residual flux is non-zero also for strong absorption lines

e.g. in the Sun b/a ≈ 9/4 → R = 0.44 (λ = 5000 A)

However resonance lines such as Na D have R ~ $10^{-3} – 10^{-4}$

***b) case ε = 0 (extreme non-LTE: pure scattering lines)***

$$\lambda_\nu = \frac{1}{1+\beta_\nu} \qquad R_\nu = \frac{\frac{1}{1+\beta_\nu} + \sqrt{\frac{3}{1+\beta_\nu}}a}{(1+\sqrt{\frac{1}{1+\beta_\nu}})(b+\sqrt{3}a)}$$

as $\beta_\nu \rightarrow \infty$ (strong line): $R_\nu = 0$

for a strong line formed by scattering

scattering removes all photons → no photon emerges from surface

# Theoretical curve of growth

standard diagnostic tool for the determination of metal abundances in cool stars
simple model which allows equivalent widths to be calculated analytically

***assumptions:***

1. pure absorption lines
2. Milne-Eddington model
3. LTE
4. $\varepsilon_\nu$ = 1 (no scattering)

$$R_\nu = \frac{\frac{b}{1+\beta_\nu} + \sqrt{3}a}{b + \sqrt{3}a}$$

$$\beta_\nu = \frac{\kappa_\nu^L}{\kappa^C} = \frac{n_l B_{lu} - n_u B_{ul}}{\kappa^C} \frac{h\nu}{4\pi} \varphi_\nu$$

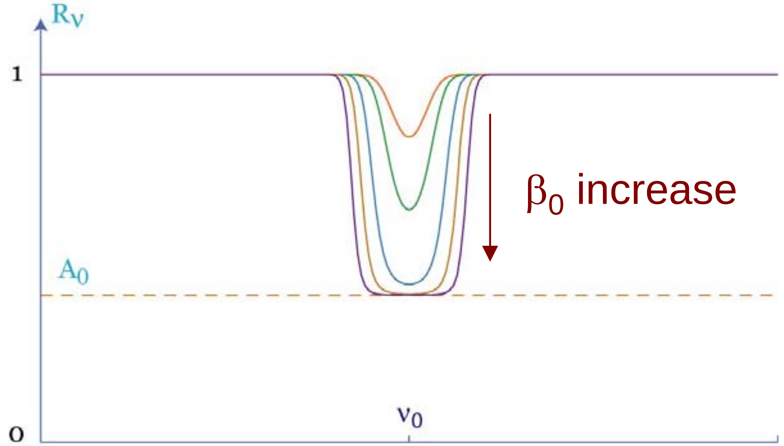
$$= \frac{n_l B_{lu}(1 - e^{-h\nu/kT})}{\kappa^C} \frac{h\nu}{4\pi} \varphi_\nu = \frac{n_l}{\kappa^C} (1 - e^{-h\nu/kT}) \frac{\pi e^2}{mc} f_{lu} \varphi_\nu = \beta_0 \varphi_\nu$$

# Theoretical curve of growth

$$R_\nu = \frac{\frac{b}{1+\beta_0\,\varphi_\nu} + \sqrt{3}a}{b+\sqrt{3}a}$$

$$A_\nu = 1 - R_\nu = \frac{\beta_0\,\varphi_\nu}{1+\beta_0\,\varphi_\nu}\left(\frac{b}{b+\sqrt{3}a}\right) = A_0\,\frac{\beta_0\,\varphi_\nu}{1+\beta_0\,\varphi_\nu}$$

line depth

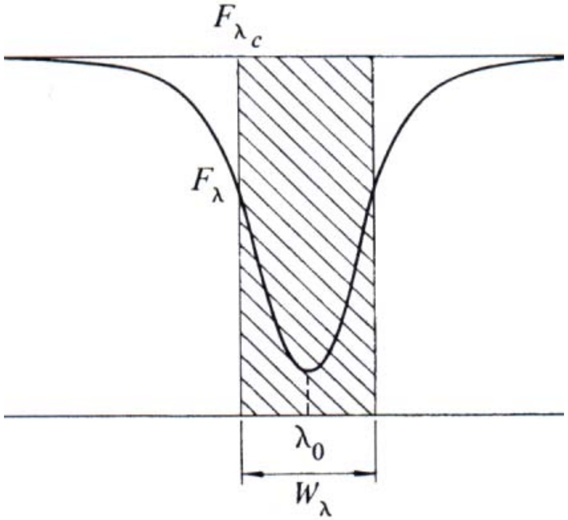



in LTE non-zero central intensity even for strong lines ($\beta \rightarrow \infty$)

# Theoretical curve of growth

**Equivalent width**

$$W_\nu = \int_0^\infty A_\nu \, d\nu$$




$$\Rightarrow \quad W_\nu = A_0 \beta_0 \int_0^\infty \frac{\varphi_\nu}{1 + \beta_0 \, \varphi_\nu} \, d\nu$$

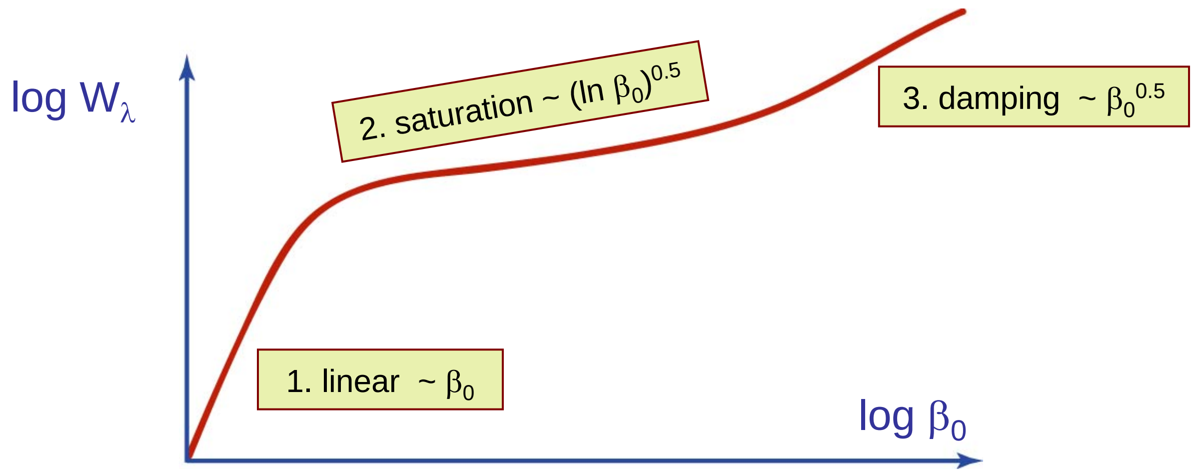
use Voigt profile

$$\varphi(\nu - \nu_0) = \frac{1}{\pi^{1/2} \, \Delta\nu_D} \; H(a, v)$$

$$H(a,v) \approx e^{-v^2} + \frac{a}{\pi^{1/2} v^2}$$

# Theoretical curve of growth

**3 regimes**




**1. linear regime:** unsaturated Doppler core $H(a,v) \approx e^{-v^2}$

$$W_\nu \approx \frac{A_0\beta_0}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{e^{-v^2}}{1+\frac{\beta_0}{\sqrt{\pi}\Delta\nu_D}}\, dv = \frac{A_0\beta_0}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-v^2}\left(1-\frac{\beta_0}{\sqrt{\pi}\Delta\nu_D}e^{-v^2}+\ldots\right) dv \approx A_0\beta_0$$

$$\frac{\beta_0}{\Delta\nu_D} < 1$$

# Theoretical curve of growth

**2. saturation part:** core saturation but line wings not important

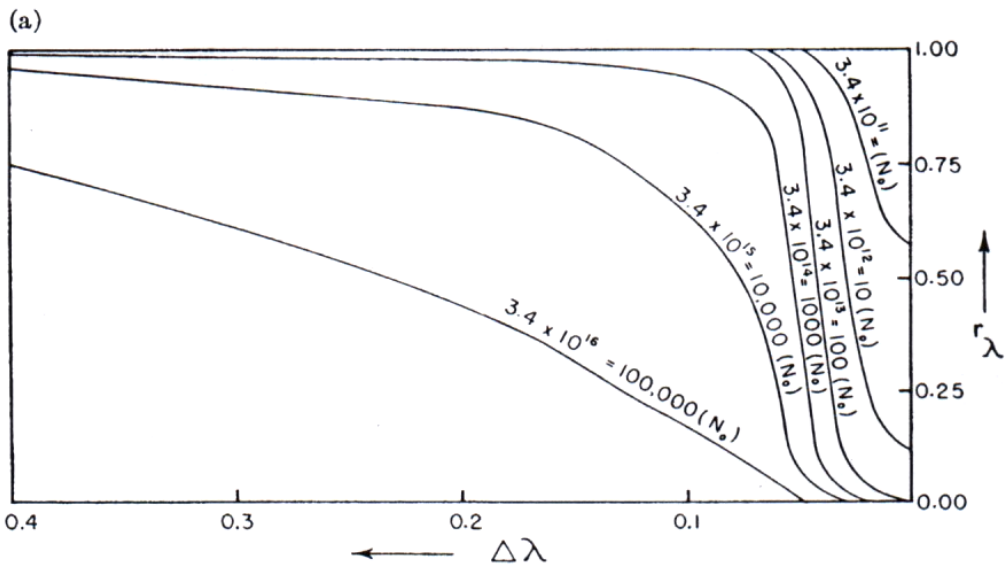
as in previous case but $\frac{\beta_0}{\Delta\nu_D} > 1$

$$W_\nu \approx 2A_0\Delta\nu_D\sqrt{ln[\beta_0/(\sqrt{\pi}\Delta\nu_D)]} \sim \sqrt{ln\beta_0}$$

**3. damping part:** line wings dominate $H(a,v) \approx \frac{a}{\pi^{1/2}v^2}$

$$W_\nu \approx \frac{A_0\beta_0}{\pi}a\int_{-\infty}^{\infty}\frac{1}{v^2+\frac{\beta_0 a}{\pi\Delta\nu_D}}\,dv = A_0\left(a\pi\Delta\nu_D\beta_0\right)^{1/2}$$

# Theoretical curve of growth




*effect on a spectral line of the increase of absorbers along the line of sight*

# Theoretical curve of growth

In general: $W_\nu = f(\beta_0)$ or $f\left(\frac{\beta_0}{\Delta\nu_D\sqrt{\pi}}\right) = f(\beta^*)$

$$\beta^* = \frac{\pi e^2}{mc} f_{lu} \frac{n_l}{\kappa^C} (1 - e^{-h\nu/kT}) \frac{1}{\Delta\nu_D\sqrt{\pi}}$$

$$\kappa_\nu^c = \kappa_0^c (1 - e^{-h\nu/kT})$$ in LTE

$$n_l = n_1 \frac{g_l}{g_1} (1 - e^{-h\nu/kT})$$ Boltzmann

$$\Delta\nu_D = \frac{\nu_0}{c}\sqrt{\frac{2kT}{m}} = \frac{1}{\lambda}\sqrt{\frac{2kT}{m}}$$

# Theoretical curve of growth

$$\log \beta^* = \log(g_l f_{lu}\lambda) + \log e^{-E_{1l}/kT} + \log\left(\frac{n_1}{g_1\kappa_0^c}\frac{\sqrt{\pi}e^2}{mc}\sqrt{\frac{m}{2kT}}\right)$$

$$\log \beta^* = \log(g_l f_{lu}\lambda) - \frac{5040 E_{1l}}{T} + \log C$$

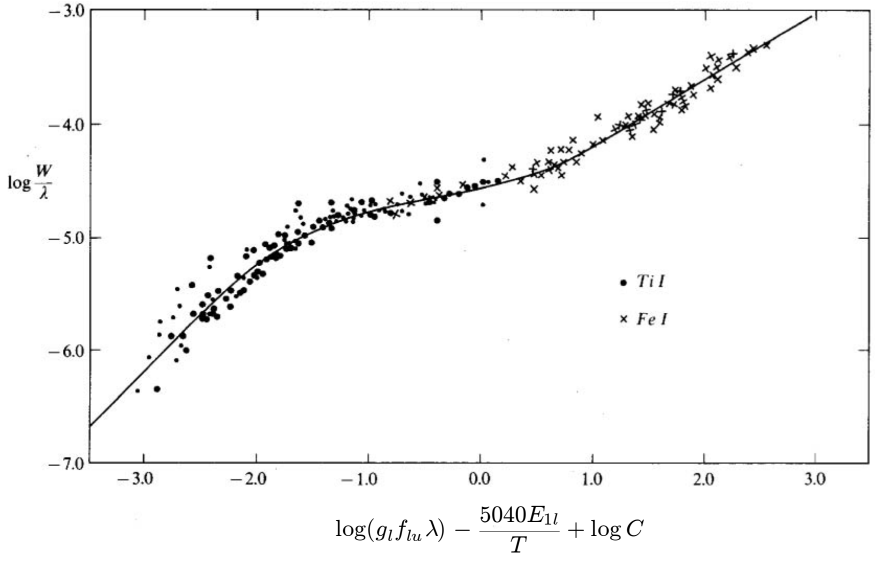
1. for a single ionization stage C = const
2. lines belonging to one ionization stage should form a curve of growth: $\beta^*$ varies as a function of line transition
3. if T and $\kappa_0$ known: shift observed $W_\nu$ curve until it matches theoretical curve
4. from $n_1$ calculate total abundances using Saha-Boltzmann equations

# Theoretical curve of growth

**empirical curve of growth**

plot *log (W/λ)* vs *log (gfλ) - 5040* $E_{1l}/T$ for each line

adjust T to minimize scatter around a mean curve (excitation temperature typical of line formation region)




*empirical curve of growth for iron and titanium lines in the Sun*

Mihalas, 78