1. For the hydrogen line H$_{\delta}$ (transition 2 $\rightarrow$ 6) the line broadening profile can be approximated by

$$\varphi(\Delta \lambda) = \varphi_D(\Delta \lambda) = \pi^{-0.5} \Delta \lambda_D^{-1} \exp\left(-\left(\frac{\Delta \lambda}{\Delta \lambda_D}\right)^2\right)$$

for $|\Delta \lambda| \leq \Delta \lambda_t$

$$= \varphi_S(\Delta \lambda) = F_0^{1.5} K \Delta \lambda^{-2.5}$$

for $|\Delta \lambda| \geq \Delta \lambda_t$

$\varphi_D(\Delta \lambda)$ is the thermal Doppler broadening in the line core with

$$\Delta \lambda_D = 4.2846 \times 10^{-7} \lambda_0 T^{0.5}, \lambda_0 = 4101.73 \text{ Å.} T \text{ is the temperature in K.}$$

$\varphi_S(\Delta \lambda)$ is the Stark broadening in the line wing with $K = 0.01944$ and

$$F_0 = 1.25 \times 10^{-9} n_e^{2/3}, \text{ where the electron density } n_e \text{ is in cm}^{-3}.$$ 

$\Delta \lambda = \lambda - \lambda_0$ is the wavelength displacement from the line center in Å.

$\Delta \lambda_t$ is the $\Delta \lambda$ – value, where $\varphi_D(\Delta \lambda) = \varphi_S(\Delta \lambda)$. 
a) Calculate and plot $\log [\varphi (\Delta \lambda)]$ for $T = 10,000 K$
and $n_e = 10^{10}, 10^{11}, 10^{12}, 10^{13} \text{ cm}^{-3}$.

(3 points)

b) Assume a plane-parallel layer of constant temperature $T$, constant electron density $n_e$, constant line absorption coefficient, and constant line source function $S(\Delta \lambda)$. The incident intensity $I_0$ is independent of wavelength and the ratio of $S(\Delta \lambda)/I_0 = 0.2$ (also independent of wavelength). The optical thickness in the center of the line ($\Delta \lambda = 0$) is $\tau_0 = 10^4 \pi^{-0.5} \Delta \lambda_D^{-1}$. Solve the equation of radiative transfer for the full line profile under these assumptions and plot the line profile intensity $I(\Delta \lambda)/I_0$ emerging from the layer for $T = 10,000 K$ and $n_e = 10^{10}, 10^{11}, 10^{12}, 10^{13} \text{ cm}^{-3}$.

(3 points)

Please return homework on Tuesday, March 2.