1. For the hydrogen line H_δ (transition 2 → 6) the line broadening profile can be approximated by

\[ \varphi(\Delta \lambda) = \varphi_D(\Delta \lambda) = \pi^{-0.5} \Delta \lambda_D^{-1} \exp\left(-\frac{\Delta \lambda}{\Delta \lambda_D}\right)^2, \text{ for } |\Delta \lambda| \leq \Delta \lambda_t \]

\[ = \varphi_S(\Delta \lambda) = F_0^{1.5} K \Delta \lambda^{-2.5}, \text{ for } |\Delta \lambda| \geq \Delta \lambda_t \]

\( \varphi_D(\Delta \lambda) \) is the thermal Doppler broadening in the line core with

\[ \Delta \lambda_D = 4.2846 \times 10^{-7} \lambda_0 \ T^{0.5}, \lambda_0 = 4101.73 \ \text{Å}. \ T \text{ is the temperature in K.} \]

\( \varphi_S(\Delta \lambda) \) is the Stark broadening in the line wing with \( K = 0.01944 \) and

\[ F_0 = 1.25 \times 10^{-9} n_e^{2/3}, \text{ where the electron density } n_e \text{ is in cm}^{-3}. \]

\( \Delta \lambda = \lambda - \lambda_0 \) is the wavelength displacement from the line center in Å.

\( \Delta \lambda_t \) is the \( \Delta \lambda - \) value, where \( \varphi_D(\Delta \lambda) = \varphi_S(\Delta \lambda) \).
a) Calculate and plot \( \log[\varphi(\Delta\lambda)] \) for \( T = 10,000K \) and \( n_e = 10^{10}, 10^{11}, 10^{12}, 10^{13} \) cm\(^{-3}\).

(3 points)

b) Assume a plane-parallel layer of constant temperature \( T \), constant electron density \( n_e \), constant line absorption coefficient, and constant line source function \( S(\Delta\lambda) \). The incident intensity \( I_0 \) is independent of wavelength and the ratio of \( S(\Delta\lambda)/I_0 = 0.2 \) (also independent of wavelength). The optical thickness in the center of the line \( (\Delta\lambda=0) \) is \( \tau_0 = 10^4 \pi^{-0.5} \Delta\lambda_D^{-1} \). Solve the equation of radiative transfer for the full line profile under these assumptions and plot the line profile intensity \( I(\Delta\lambda)/I_0 \) emerging from the layer for \( T = 10,000K \) and \( n_e = 10^{10}, 10^{11}, 10^{12}, 10^{13} \) cm\(^{-3}\).

(3 points)

Please return homework on Thursday, April 3.