Einstein's theory of General Relativity

- light rays deflected by gravity
- first test: sun → predicted reflection 1.7" confirmed to 0.1%

calculation of deflection angle

1. light travels form source to a point close to lens through unperturbed space time with velocity $v_1 = c$
2. near lens light is deflected through the gradient of the gravitational potential which adds a vertical velocity component $v_z$
Newtonian approximation
(light as a particle)

\[ \alpha = \frac{v_z}{c} = \frac{1}{c} \int \frac{d\Phi}{dz} \, dt \]

\[ = \frac{1}{c^2} \int \frac{d\Phi}{dz} \, dl \]

\( \Phi = \text{grav. potential of lens} \)

acceleration in z-direction

Johann von Soldner, 1804
Munich University Observatory

General Relativity

\[ \vec{\alpha} = \frac{2}{c^2} \int \vec{\nabla}_\perp \Phi \, dl \]

differs only by factor of two

\[ \vec{\alpha} \]

vector determined by integral of potential gradient perpendicular to light propagation
point mass

$$\Phi = -GM \frac{1}{(l^2 + z^2)^{\frac{1}{2}}} \quad \vec{\nabla}_\perp \Phi = \frac{d\Phi}{dz} = GM \frac{z}{(l^2 + z^2)^{\frac{3}{2}}}$$

$$\alpha(z) = \frac{2}{c^2} GM \int_{-\infty}^{\infty} \frac{z}{(l^2 + z^2)^{\frac{3}{2}}} \, dl = \frac{4}{c^2} GM z \int_{0}^{\infty} \frac{dl}{(l^2 + z^2)^{\frac{3}{2}}}$$

$$\alpha(z) = \frac{4}{c^2} GM z \left[ \frac{l}{(l^2 + z^2)^{\frac{1}{2}}} \right]_0^{\infty} = \frac{4}{c^2} GM \frac{1}{z}$$

at light ray impact parameter $b = z$ \Rightarrow

$$\alpha = 2 \frac{R_s}{b}$$

Schwarzschild radius

$$R_s = 2G \frac{M}{c^2}$$

for the sun: $R_s = 3.0$ km, $b = R_{\text{sun}} \Rightarrow \alpha = 1.7''$
lens equation

note: all distances are angular diameter distances and, thus, $D_{ds} \neq D_s - D_d$

$\Theta$ angle between source and lens caused by the effect of lensing
$\beta$ angle between source and lens without effect of lensing
$\alpha$ Einstein deflection angle

important relations:

$$\alpha D_{ds} = (\Theta - \beta) D_s$$

$$\Theta D_s = \beta D_s - (\Theta - \beta) D_s$$

$$\beta = \Theta - \frac{D_{ds}}{D_s} \alpha$$

relates real position (angle) of the source without the action of lens with the position of the lensed image
for a point mass

\[ \Theta^2 - \Theta \beta - \Theta^2_E = 0 \]

\[ \Theta^2_E = 2R_S \frac{D_{ds}}{D_s D_d} \]

lens equation for angular position of lensed image

angular position Einstein radius

point mass lensed images

1. \( \beta = 0 \rightarrow \) observer, lens, source aligned

\[ \Theta = \Theta_E \]

Einstein ring
diameter determined by mass \( M \) because

\[ R_S = 2G \frac{M}{c^2} \]

2. \( \beta \neq 0 \rightarrow 2 \) solutions

\[ \Theta_1 = \frac{\beta}{2} + \sqrt{\frac{\beta^2}{4} + \Theta^2_E} \]

\[ \Theta_2 = \frac{\beta}{2} - \sqrt{\frac{\beta^2}{4} + \Theta^2_E} \]

< 0 \rightarrow on other side of lens

separation depends on \( \Theta_E \) and, thus, on mass of lens
extended mass distributions

lensing galaxies are no point masses \( \rightarrow \) next level of approximation

- lens infinitely thin mass sheet perpendicular to line of sight
- plane of mass sheet = lens plane
- surface mass density
  \[
  \Sigma(\vec{R}) = \int_{\Delta l} \rho(\vec{r}, \vec{l}) dl
  \]

the deflecting (vector) angle is then

\[
\vec{\alpha}(\vec{R}) = \frac{4G}{c^2} \int \frac{(\vec{R} - \vec{R}')\Sigma(\vec{R}')}{|\vec{R} - \vec{R}'|^2} d^2 \vec{R}'
\]

and the lens equation becomes

\[
\vec{\beta} = \vec{\Theta} - \frac{D_{ds}}{D_s} \vec{\alpha}(\vec{\Theta})
\]

with the possibility of multiple images as solutions
spherical mass distributions

for instance, elliptical galaxies with an isothermal Jaffe-model

\[ \rho(r) = \frac{M}{4\pi r_0^3} \left( \frac{r}{r_0} \right)^2 \left( 1 + \frac{r}{r_0} \right) \]

\[ \Phi(r) = \frac{GL}{r_0} \left( \frac{M}{L} \right) \ln \frac{r}{r + r_0} \]

have

\[ \alpha(R) = 4G \frac{M(r)}{c^2 R} \quad \text{with} \quad M(R) = 2\pi \int_0^R \Sigma(R')R'dR' \]

and the lens equation becomes

\[ \beta = \Theta - \frac{D_{ds}}{D_s D_d} \frac{4G}{c^2} \frac{M(D_d \Theta)}{\Theta} \]

or

\[ \Theta^2 - \Theta \beta - \frac{D_{ds}}{D_s D_d} \frac{4G}{c^2} M(D_d \Theta) = 0 \]
light from background source (QSO) lensed by foreground galaxy G → multiple images A, B, C, D, S

1. QSO is variable with light curve
   → A, B, C, D, S also show the QSO light curve

2. light paths from source via A, B, C, D, S different
   → time delays between A, B, C, D, S light curves

3. the gravitational lens equation allows to construct a model of the galaxy spatial mass distribution
   → the angular separation between images A, B, C, D, S and galaxy G can be turned into an absolute length $l_{\text{lense}}$ which will be of the order

   \[ l_{\text{lense}} \approx \Theta_E D_d \]

→ the angular separation of images and the absolute length $l_{\text{lense}}$ can then be used to determine the angular distance $D_d$ and, thus, the Hubble constant $H_0$
simple estimate of time delay time scale

light travel time $t(z)$ from source to earth is given by formula in chapter 15

$$t(z) = \frac{1}{H_0} \int_0^z \frac{dz}{(1 + z)E(z)}$$

the lens causes a “detour” approximated here in Euclidean geometry

assume $D_{ds} \approx D_d$ for simplicity

$\Delta d \approx D_d \frac{1}{2} \frac{\Theta_E^2}{D_d}$

$\Theta_E^2 = 2R_S \frac{D_{ds}}{D_s D_d} \approx R_S \frac{1}{D_d}$

$\Delta d \approx R_S$

$\Delta t \approx \frac{R_S}{c}$

$M_{\text{galaxy}} \approx 10^{12} M_{\text{sun}} \rightarrow R_S \approx 3 \times 10^{12}$ km $\rightarrow \Delta t \approx 10^7$ sec $\approx 0.3$ yrs

in an accurate calculation one needs to account for detailed light travel through G.R. space time !!!
Gravitational Lens Time Delays

RXJ1131-1231

Time delay:
\[ t = \frac{1}{c} D_{\Delta t} \phi_{\text{lens}} \]

Time-delay distance:
\[ D_{\Delta t} \propto \frac{1}{H_0} \]

Obtain from lens mass model

For cosmography, need:
(1) time delays
(2) lens mass model
(3) mass along line of sight

Advantages:
- simple geometry & well-tested physics
- one-step physical measurement of a cosmological distance

2014 Miapp WS
Brief History

• 1964: Method proposed by Refsdal
• 1970s: First lenses discovered
• 1980s: First time delay measured
  • Controversy. Solution: improve sampling
• 1990s: First Hubble Constant measured
  • Controversy. Solution: improve mass models
• 2000s: modern monitoring (COSMOGRAIL, Fassnacht, Kochanek & others)
• 2010s: Putting it all together.
  Advances in 1) time delays
    2) lens mass model
    3) mass along line of sight
  precision measurements (5-8% from a single lens)
(1) Time Delays

\[ t = \frac{1}{c} D_{\Delta t} \phi_{\text{lens}} \]

Dedicated monitoring

- Radio
  [e.g., Fassnacht et al. 2002]
- Optical
  COSMOGRAIL
  [e.g., Courbin et al. 2011, Tewes et al. 2013]
  Kochanek et al. 2006

[Vanderriest et al. 1989]

S. Suyu, 2014/05/29
(1) Time Delays

- monitoring lensed quasars since 2004 in the optical
- expect to have delays with a few percent error for ~20 lenses

EPFL: **G. Meylan, F. Courbin**, M. Tewes, C. Faure, Y. Revaz, N. Cantale
(Formerly: A. Eigenbrod, C. Vuissoz)
IIA Bangalore: T. Prabhu, C.S. Stalin, R. Kumar, D. Sahu
Univ. Bonn: D. Sluse
Univ. Liège: P. Magain, E. Eulaers, V. Chantry
UzAS Tashkent: I. Asfandiyarov
Univ. Zürich: P. Saha, J. Coles
Univ. Nottingham: S. Dye

Now also in close collaboration (monitoring, microlensing) with:
C. Kochanek, A. Mosquera (Ohio), C. Morgan, C. MacLeod, L. Hainline (USNA)
(1) Time Delays

**RXJ1131:**
Time delay with 1.5% Accuracy!

[Tewes et al. 13b]

Based on state-of-the-art curve modeling techniques
[Tewes et al. 13a, Hojjati et al. 13]
The Time-Delay Challenge

TDC0 (Dobler et al. 2013): what does it take to infer time-delays?
Gravitational Lens Time Delays

RXJ1131-1231

Time delay:

\[ t = \frac{1}{c} \frac{D_{\Delta t}}{\phi_{\text{lens}}} \]

Time-delay distance:

\[ D_{\Delta t} \propto \frac{1}{H_0} \]

Obtain from lens mass model

For cosmography, need:

✓ (1) time delays
✓ (2) lens mass model
✓ (3) mass along line of sight
(2) Lens Mass Model

\[ t = \frac{1}{c} D_{\Delta t} \phi_{\text{lens}} \]

Use extended images of AGN host

Used only image positions of AGN (providing few constraints)

[e.g., Dye et al. 2005, Suyu et al. 2009]
(2) Lens Mass Model

light distribution of extended source

mass distribution of lens

light of lens
(Sersic)

light of lensed AGN
+ time delays
(2) Lens Mass Model
(3) Mass along Line of Sight

Lens Only

Distance measurement changes

Lens with structures along line of sight

\[ D_{\Delta t}^{\text{true}} = \frac{D_{\Delta t}^{\text{model}}}{1 - \kappa_{\text{ext}}} \]
(3) Mass along Line of Sight

galaxy number counts + Millennium Simulation

Stellar kinematics of lens further constrain $\kappa_{\text{ext}}$

[Suyu et al. 2010]

$z_S = 1.39$

$n_{\text{gal}}/\langle n_{\text{gal}} \rangle = 2 \pm 0.05$

all l.o.s.

Ken Wong’s talk for more recent developments
**H₀ from Time Delay Lenses**

ALL mass along the line of sight (LOS) affects the lens

\[ \Delta t \propto D \Delta t \phi_{lens} \]

\[ D_{true}^{\Delta t} = \frac{D_{model}^{\Delta t}}{1 - \kappa_{ext}} \]

\( \kappa_{ext} \) comes from LOS mass distribution

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Kenneth Wong (ASIAA)
Lens Environment Effects on H₀

MIAPP - 6/4/2014

Figure Courtesy of LSST
Mass Along the Line of Sight

- External mass can bias Δt measurements → bias on $H_0$ (dominant source of uncertainty)
- Also affects other distance indicators (SN Ia, gravitational wave standard sirens)
- Lens models only account for external shear, $\gamma$
- Recent studies quantify LOS effects on $H_0$ with external convergence in lens plane, $K_{\text{ext}}$ (e.g. Suyu et al. 2010, 2013)
- $K_{\text{ext}}$ is statistical correction calibrated to simulations using galaxy number counts (Fassnacht et al. 2011)
- If we know where mass is along LOS, we can include it explicitly in the mass model

Kenneth Wong (ASIAA)
Lens Environment Effects on $H_0$

The Extragalactic Distance Scale
MIAPP - 6/4/2014
Multi-Plane Lensing

- We only observe the images after final deflection (i = 1 lens plane)
- Each lens plane acts on lights rays that have already been deflected by all previous planes - non-linear effects
- Computation speed slows dramatically with many perturbers
- Lenses can have hundreds of galaxies projected within ~few arcminutes, need to be accounted for

McCully+2014

Treu (2010)

Kenneth Wong (ASIAA)
Lens Environment Effects on H₀

The Extragalactic Distance Scale
MIAPP - 6/4/2014
New Multi-Plane Lensing Formalism
(see McCully et al. 2014; arXiv:1401.0197)

- Some perturbers are more important than others, need to be included explicitly - “main planes”
- Most perturbers can be approximated by 2nd order terms in lens potential (shear/convergence) - “tidal planes”
- Can isolate effect of tidal planes alone - only need to compute once!
Characterizing the environment

Figure: Ray-tracing through the lightcone of the lens (McCully et al. 2014, Collett et al. 2013).
Where are Perturbers Most Significant?

- Run tests with mock lens at $z_L = 0.3$, source at $z_S = 2$, input $H_0 = 71$
- Point mass perturber, model lens using tidal approximation for perturber
- Smaller angular offset $\rightarrow$ bigger effect on recovered $H_0$
- Perturber in foreground seems to be more important than in background...

Kenneth Wong (ASIAA)
Lens Environment Effects on $H_0$
Two time-delay lenses

B1608+656

Discovery: Myers et al. 1995
Delays: Fassnacht et al. 2002

Pilot Study:
measured $D_{\Delta t}$ to 5%

RXJ1131-1231

Discovery: Sluse et al. 2003
Delays: Tewes et al. 2013b
Modeling: Suyu et al. 2013

Blind Analysis:
test analysis method
How to get more lenses?

Search through current and future imaging surveys!

Two ongoing large imaging surveys:
• Dark Energy Survey
  [The STRong-lensing Insights into Dark Energy Survey]
  ➔ Adriano Agnello’s talk
• Hyper Suprime-Cam Survey

S. Suyu, 2014/05/29
To make this possible, we have formed a broad external collaboration of DES: STRong lensing Insight into DES (STRIDES). We expect $\sim 10^3$ lensed quasars in the DES footprint, $\sim 120$ with $i < 21$.

strides.physics.ucsb.edu
Figure: STRIDES around the world: QSO selection; lensed QSO candidate selection; imaging (confirmation and science); time-variability and time-delay monitoring; spectroscopic follow-up; modelling.
Hyper Suprime-Cam (HSC) Survey

collaboration: Japan, Princeton, Taiwan
PI: Satoshi Miyazaki

116 CCD chips
(870 million pixels)
HSC Survey

Wide: 1400 deg$^2$, grizy, $r\sim26$
Deep: 27 deg$^2$, grizy+3NB, $r\sim27$
Ultra-Deep: 3.5 deg$^2$ (2 pointings), grizy+3NB, $r\sim28$
Lensed quasars from the ground

How to tell if this is a lens?

use configuration of blended images

[Anguita et al. 2009]

S. Suyu, 2014/05/29
Simulated Lens

Simulated lens in CFHTLS by Anupreeta More

CHITAH classifies this as a lens!!

[Chan, Suyu et al., in prep]
Lens object in CFHT?

Object in CFHTLS

CHITAH classifies this as NOT a lens!

[Chan, Suyu et al., in prep]
SHARP: An alternative approach

- SHARP = Strong-lensing High Angular Resolution Program
- Use laser guide star adaptive optics with Keck II Telescope
- Get resolution comparable to or better than HST, while using a mirror that has 16 times the collecting area

- Team SHARP:
  Chris Fassnacht, Simona Vegetti,
  John McKean, Dave Lagattuta,
  Leon Koopmans, Matt Auger

\[ \theta \sim \frac{\lambda}{D} \]
AO vs. Space: B0712+472

F555W  
F814W  
F160W

F160W, again

Keck AO K’-band

z_l = 0.41

[Fassnacht et al. in prep]  [Material courtesy of Chris Fassnacht]
SHARP for time-delay cosmography?

Can AO data produce similar cosmological constraints as HST data for time-delay systems?

First test on RXJ1131-1231 [Chih-Fan Chen et al, in preparation]

Challenge: Unknown Point-Spread Function (PSF) of AO

HST/ACS F814W  Keck AO Ks (1 hr exposure)
Future Prospects

- Current and future surveys will yield hundreds of new QSO lenses [Oguri & Marshall 2010]:
  - HSC: ~600 (~80 quads)
  - DES: ~1000 (~130 quads)
  - LSST: ~8000 (~1000 quads)

When combine 150 lenses with CMB+SN:
- Area of $w_a - w_0$ contour tightens by a factor of ~5
- All cosmological parameters are better determined by factors of ~3 with the inclusion of time delays

Time-delay lenses are excellent complements to other probes
Linder (2011): When time-delay lensing data are exploited jointly with supernovae and CMB information, this enables the accurate measurement of the matter density $\Omega_m$ (to within 0.004), the Hubble constant $h$ (to within 0.7%) and the variation of the dark energy equation of state $w_a$ (to within 0.26). This requires a careful study of the systematics and a large enough sample of lenses.
Summary

• Time-delay distances $D_{\Delta t}$ of each lens can be measured with uncertainties of ~5-8% including systematics
• Blind analysis of RXJ1131-1231 demonstrated robustness of method
• Lenses highly complement other probes
• Stay tuned for results from H0LiCOW
• Robot development is underway to find new lenses in HSC
• Work is in progress to test whether AO is viable for follow up
• Current and future surveys will find at least hundreds of time-delay lenses, providing an independent and competitive probe of cosmology