5. The Baade-Wesselink Method

Baade-Wesselink Method (BWM): distance determination method for stellar objects, for which the photosphere can be approximated as a spherically moving (expanding) shell.

\[ \Delta R = R(t) - R(t_0) = \int_{t_0}^{t} v(t) \, dt \]

change of radius determined by integral of velocity
radius $R$, angular diameter $\Theta$, and distance $d$ are related

$$\frac{R}{d} = \tan \frac{\Theta}{2} \approx \frac{\Theta}{2}$$

change in radius $\Delta R$
change in angular diameter $\Delta \Theta$

measure $\Delta R$ from velocity
and find a way to measure $\Delta \Theta \rightarrow$ distance
original idea: test pulsation hypothesis of Cepheids

- Walter Baade,
  1926, Astronomische Nachrichten 228, 359
  “Über eine Möglichkeit, die Pulsationstheorie der δ Cephei-Veränderlichen zu prüfen”

- Adriaan Jan Wesselink,
  1946, Bulletin of the Astronomical Institutes of the Netherlands 10, 91
  “The observations of brightness, colour and radial velocity of δ Cephei and the pulsation hypothesis”

later application: calibrate absolute magnitudes of radial pulsators and supernovae
Baade-Wesselink distances

Angular diameter from interferometry, or surface-brightness relation (i.e. lightcurve in colour)

M. Groenewegen
measurement of $\Delta \Theta = 2 \Delta R/d$

method 1: measure stellar surface flux $\pi F_{\lambda}$ and stellar brightness $S_{\lambda}$

$$S_{\lambda}(t) = \frac{R^2(t)}{d^2} \pi F_{\lambda}(T_{eff}(t))$$

changes $\Delta S_{\lambda} \rightarrow \Delta \Theta$, $F_{\lambda}(t)$ from spectral analysis and model atmospheres or surface brightness-color relationship of G,K,M giants
Surface Brightness Color Relationship

Interferometry → obs. surface brightness color relationship

SBCR
late type giants G, K, M

σ ~ 0.03 mag

DiBenedetto, 1998, 2005
Groenewegen, 2004
Kervella, 2004
BWM distances

\[ SB[\text{mag}] = 2.656 + 1.483(V-K)_0 + 0.044(V-K)_0^2 \]

\[ \varphi[\text{mas}] = 10^{0.2(SB - V)} \]

\[ d[\text{pc}] = 9.305 \frac{\Delta R[R_{\odot}]}{\Delta \varphi[\text{mas}]} \]

SBCR

\[ \sigma = 0.03 \text{ mag} \]

angular diameter

distance
method 2: interferometry

Interferometric observations of δ Cep (Mérand et al. 2005)
simple straightforward method, but the devil is in the detail....

radial velocity from spectral lines ≠ radial expansion velocity \( v(t) \)

1. \( v_{rad} \) is integral over stellar surface

\[
v(t) = pv_{rad}(t)
\]

projection factor \( p \), \( p > 1 \)

2. hydrodynamic motion of matter caused by pulsation

\( \rightarrow \) very likely velocity gradients in photosphere
The Baade-Wesselink methods

Interferometry

or

IRSB

\[ d \propto \frac{\Delta R}{\Delta \theta} \text{ with } R(t) = \int p V_{\text{rad}} dt \]

\[ \Delta R \Leftrightarrow \Delta \theta \]

A direct linear relation


or

\[ p = \frac{V_{\text{puls}}}{V_{\text{rad}}} \]
The radial velocity definition

$\beta$ Dor HARPS observations
The radial velocity definition

- **A geometric effect (uniform disk)**

  \[ V_{\text{puls}} = 30\text{km/s} \]

  \[ RV_c = 20\text{km/s} \]

  \[ p = 1.5 \]

- **Limb-darkening effect**

  \[ V_{\text{puls}} = 30\text{km/s} \]

  \[ RV_c = 21.5\text{km/s} \]

  \[ p = 1.39 \]

  \[ p_0 = -0.18uV + 1.50 \]

  \[(\text{Nardetto et al. 2006})\]
HARPS observations of 10 Cepheids (P=3j à P=42j)

300 spectra
Thousands of spectral lines
17 selected

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<th>Wavelength (Å)</th>
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$\Delta RV_c = a_o D + b_o$
The p-factor decomposition

- Direct measure of the velocity gradient (HARPS)
- Geometric p-factor
- Extrapolation to the photosphere
- Optical/gas layers: hydro code

\[
p = 1.39 \times 0.99 \times 0.96 = 1.33
\]
The Pp relation

High resolution spectroscopy for Cepheids distance determination II. A period-projection factor relation

\[ p = [−0.06 \pm 0.02] \log P + [1.38 \pm 0.02] \]
And when using the cross-correlation method?

High resolution spectroscopy for Cepheids distance determination of the cross-correlation on the $p$-factor and the $c$-velocity

V. Impact


$5\%$

$\rho = -[0.08 \pm 0.05] \log P + [1.31 \pm 0.06]$

Conclusions:

1 - the static approach lead to an overestimation of the distances of 10%
2 - consistent with HST parallaxes measurements Mérand et al. 2005; Groenewegen 2007
3 - slope PL (Milky Way) = slope PL (LMC) P. Fouqué et al., 2007, A&A, 476, 73
An Example

$V, K, \text{RV phased light curves for AQ Pup}$
\( \theta \) vs. \( \Delta R \) gives distance. AQ Pup
Distance to the LMC should be independent of Period
Distance to the LMC should be independent of (mean) \((V - K)\) colour
Slope of the \(M_V\) or \(K - P\) relation should be the same as the \(m_V\) or \(K - P\) one

LMC distance (corrected for "plane") versus Period for \(p = 1.33\)
$p(P)$ dependence

\[ 1.50 - 0.24 \log P = p \]

\[ d_{LMC} = 45.5 \text{ kpc} \]

Groenewegen

MIAPP, 17 June 2014 – p.23/50
For the LMC sample we have expanded the sample to 36 stars by obtaining radial velocity curves for 22 LMC Cepheids based on observations with the HARPS spectrograph on the ESO 3.6m telescope. (Storm et al. 2011)
● LMC photometry
  ● Persson et al. (2004)
Radial velocity curves for galactic Cepheids using the 1.2m STELLA-I robotic telescope on Tenerife with the SES fiber-fed echelle spectrograph. (Storm et al. 2011)
The new constraints on the p-factor, $p = \alpha\log(P) + \beta$

Best estimate: $p = 1.550 - 0.186 \log(P)$
Comparison of distance moduli from HST parallaxes (Benedict et al. 2007) and the IRSB technique as a function of \( \log(P) \). The dispersion is 0.14 mag.
The distance to the LMC is measured to be \((m - M)_{0} = 18.45(\pm 0.04)\)
results BWM

Nardetto et al., 2009

\[ p = 1.38 - 0.06 \log P \]
\[ (m - M)_{LMC} = 18.26 \pm 0.02 \text{mag} \]

Groenewegen, 2014

\[ p = 1.50 - 0.24 \log P \]
\[ (m - M)_{LMC} = 18.29 \pm 0.02 \text{mag} \]

Storm et al., 2014

\[ p = 1.55 - 0.19 \log P \]
\[ (m - M)_{LMC} = 18.45 \pm 0.04 \text{mag} \]

10% uncertainty
Richard Anderson, 2014
BWM error source: irregularities in $v_{\text{rad}}$

$$d = \frac{2 \Delta R(t)}{\Delta \theta(t)}$$

$$\Delta d / d_1 = \frac{\Delta R_1 - \Delta R_2}{\Delta R_1} \approx 7\%$$

Independent of $p$-factor
Modulation!

\[ \Delta d/d_1 \approx 15\% \]

\[ N = 125 \]

QZ Nor, $P = 3.79\text{d}$

Anderson, 2014 MIAPP WS
Cycle-to-cycle Modulation in $l$ Car

$\Delta d/d_1 \approx 6\%$

$\ell$ Car, $P = 35.55\text{d}$

Anderson, 2014 MIAPP WS
RS Pup: an Extreme Case

$\Delta d/d_1 \approx 7\%$

RS Pup, $P = 41.51 \text{d}$

Anderson, 2014 MIAPP WS
ΔΘ Modulation?

- Yes in space-based photometry
- Interferometry? (1-2% precision required)
- ΔR and ΔΘ must be observed contemporaneously (similar puls. cycle)

Anderson, 2014 MIAPP WS