The Mauna Kea Observatories Near-Infrared Filter Set. III. Isophotal Wavelengths and Absolute Calibration

A. T. Tokunaga

Institute for Astronomy, University of Hawaii, 2680 Woodlawn Drive, Honolulu, HI 96822; tokunaga@ifa.hawaii.edu

AND

W. D. Vacca

Stratospheric Observatory for Infrared Astronomy/Universities Space Research Association, NASA Ames Research Center, MS 144-2, Moffett Field, CA 94035; wvacca@mail.arc.nasa.gov

Received 2005 January 9; accepted 2005 February 4; published 2005 March 24

ABSTRACT. The isophotal wavelengths, flux densities, and AB magnitudes for Vega (α Lyr) are presented for the Mauna Kea Observatories near-infrared filter set. We show that the near-infrared absolute calibrations for Vega as determined by Cohen et al. and Mégeissier are consistent within the uncertainties, so that either absolute calibration can be used.

1. INTRODUCTION

Simons & Tokunaga (2002) and Tokunaga et al. (2002) defined a 1–5 μm filter set that is designed to maximize sensitivity while minimizing the effects of atmospheric absorption on reducing the signal-to-noise ratio. This filter set is intended to provide good transformation between observatories located at altitudes of 2–4 km. Filter production runs have been organized to produce these filters, which are in use at more than 30 institutions. This filter set provides greater transmission than those advocated by Young et al. (1994) and was designed to provide nearly ideal photometric accuracy. To distinguish this filter set from others, we refer to it as the Mauna Kea Observatories near-infrared (MKO-NIR) filter set.

In this paper, we present the isophotal wavelengths, flux densities, and AB magnitudes for Vega for the MKO-NIR filter set. We compare the absolute calibration advocated by Cohen et al. (1992) to that of Mégeissier (1995), and we show that there is no significant difference in the flux densities for Vega derived by these authors.

2. ISOPHOTAL WAVELENGTHS AND ZERO-MAGNITUDE FLUX DENSITIES

2.1. The Definition of Isophotal Wavelength

The number of photoelectrons detected per second from a source with an intrinsic spectral energy distribution \( F_\lambda \) is given by

\[
N_p = \int F_\lambda S(\lambda) / h \nu \, d\lambda
\]

\[
= \frac{1}{hc} \int \lambda F_\lambda S(\lambda) \, d\lambda,
\]

where \( S(\lambda) \) is the total system response given by

\[
S(\lambda) = T(\lambda) Q(\lambda) R(\lambda) A_{tel}.
\]

Here \( T(\lambda) \) is the atmospheric transmission, \( Q(\lambda) \) is the product of the throughput of the telescope, instrument, and quantum efficiency of the detector, \( R(\lambda) \) is the filter response function, and \( A_{tel} \) is the telescope collecting area. The system response \( S(\lambda) \) is equal to the relative spectral response (RSR) defined by Cohen et al. (2003).

If \( F_\lambda \) and \( S(\lambda) \) are both continuous and \( S(\lambda) \) is nonnegative over the wavelength interval, then from equation (2) and the mean value theorem for integration there exists a \( \lambda_{iso} \) such that

\[
F_\lambda(\lambda_{iso}) \int \lambda S(\lambda) \, d\lambda = \int \lambda F_\lambda(\lambda) S(\lambda) \, d\lambda.
\]

Rearranging this, we obtain

\[
F_\lambda(\lambda_{iso}) = \frac{\int \lambda F_\lambda(\lambda) S(\lambda) \, d\lambda}{\int \lambda S(\lambda) \, d\lambda},
\]

where \( \lambda_{iso} \) denotes the “isophotal wavelength” and \( \langle F_\lambda \rangle \) denotes...
the mean value of the intrinsic flux above the atmosphere (in units of W m\(^{-2}\) \(\mu\)m\(^{-1}\)) over the wavelength interval of the filter. Thus, \(\lambda_{\text{iso}}\) is the wavelength at which the monochromatic flux \(F_{\lambda}(\lambda_{\text{iso}})\) equals the mean flux in the passband. Hence, \(\lambda_{\text{iso}}\) and \(F_{\lambda}(\lambda_{\text{iso}})\) are the wavelength and monochromatic flux density, respectively, that best represent a broadband (heterochromatic) measurement. In addition, we choose to use isophotal wavelengths for consistency with the extensive series of papers on infrared calibration by Cohen and collaborators.

In a similar fashion,

\[
F_{\lambda}(\nu_{\text{iso}}) = \langle F_{\nu} \rangle = \frac{\int F_{\nu}(\nu)S(\nu)\nu d\nu}{\int S(\nu)\nu d\nu},
\]

where \(\nu_{\text{iso}}\) denotes the “isophotal frequency” and \(\langle F_{\nu} \rangle\) denotes the mean value of the intrinsic flux above the atmosphere (in units of W m\(^{-2}\) Hz\(^{-1}\)) over the frequency interval of the filter.

We show in Table 1 the MKO-NIR filter isophotal wavelengths for Vega. These were calculated from equation (5) using an atmospheric model of Vega computed by R. Kurucz\(^1\) with the parameters \(T_{\text{eff}} = 9550\) K, log \(g = 3.95\), \(v_{\text{turb}} = 2\) km s\(^{-1}\), \(v_{\text{rot}} = 25\) km s\(^{-1}\), and [Fe/H] = \(-0.5\). These parameters are the same as those adopted by Bohlin & Gilliland (2004). The model has a resolving power of 10\(^5\) and has been scaled to the absolute flux level of 3.46 \times 10\(^{-9}\) ergs cm\(^{-2}\) s\(^{-1}\) \(\mu\)A\(^{-1}\) at 5556 \(\lambda\), as determined by Mégessier (1995). We used ATRAN (Lord 1992) to calculate the atmospheric transmission \(T(\lambda)\) for a range of precipitable water values between 0 and 4 mm, an air mass of 1.0, and the altitude of Mauna Kea. The measured filter response curves\(^2\) given by Tokunaga et al. (2002) were used for \(R(\lambda)\) (see their Fig. 1). We assumed that the throughput term \(\Phi(\lambda)\) was a constant over the wavelength integrals. Thus, our calculations are precisely correct only for constant detector responsivity and instrumental throughput. However, Stephens & Leggett (2004) find that the variations in the detector responsivity and instrumental throughput leads to photometric variations of \(\leq 0.01\) mag. This corresponds to variations of \(\lambda_{\text{iso}}\), of \(<0.3\%\).

In accordance with the design goals of the MKO-NIR filters, variations in \(\lambda_{\text{iso}}\) as a function of precipitable water vapor were found to be very small (\(<1\%\)) over the range of water vapor values typically encountered on Mauna Kea. For the range of 0 to 4 mm of precipitable water vapor, we found variations in \(\Delta\lambda_{\text{iso}}\) of 0.000 \(\mu\)m (\(J\)), 0.001 \(\mu\)m (\(H\)), 0.012 \(\mu\)m (\(K'\)), 0.013 \(\mu\)m (\(K\)), 0.001 \(\mu\)m (\(\lambda\)), \(-0.014\) \(\mu\)m (\(L\)), and 0.001 \(\mu\)m (\(M'\)).

The methods employed here, in particular using the number of photons detected and the formulation in equation (5), which includes the wavelength terms in the integrals, are the same as those used by Cohen et al. (1992) and subsequent papers by Cohen and his collaborators. For clarity, we have explicitly presented the equations for calculating the isophotal wavelengths for photon-counting detectors. See Bessell et al. (1998) for a discussion of how photometric results differ between energy measuring detectors and photon-counting detectors.

Real spectra do not necessarily satisfy the requirements of the mean value theorem for integration, as they exhibit discontinuities. Although the mean value of the intrinsic flux is well defined, the determination of the isophotal wavelength becomes problematic because real spectra contain absorption lines, and hence the definition can yield multiple solutions. In addition, \(\lambda_{\text{iso}}\) is not an easily measured observational quantity, because it depends on knowledge of the intrinsic source flux distribution \(F_{\lambda}(\lambda)\), which is exactly what one is attempting to determine with broadband photometry. Nevertheless, for the reasons stated above, we use isophotal wavelengths in this paper. Other definitions of the filter wavelength are briefly discussed in the Appendix, for completeness.

Any definition of the filter wavelength suffers from the limitation that the spectral energy distribution of the object being observed is likely to be different from that of Vega. Since the isophotal wavelength is different for objects that have different spectral energy distributions, a correction factor is required to obtain the monochromatic magnitude at the same isophotal wavelength as Vega. Hanner et al. (1984) discuss this problem in detail for observations of comets.

### 2.2. Absolute Flux Densities for Vega

The question of the near-infrared absolute flux densities for Vega above the atmosphere has been discussed by Cohen et al. (1992) and Mégessier (1995). Cohen et al. determined their absolute calibration from a model atmosphere for Vega multiplied by the atmospheric transmission and instrument response (filters, throughput, and filter response). They did not use absolute calibration measurements by Blackwell et al. (1983) and Selby et al. (1983), because Blackwell et al. (1990) concluded that atmospheric models of Vega offered higher precision than the observationally determined absolute calibration.

---

\(^1\) See http://kurucz.harvard.edu/stars/VEGA.

in the near-infrared. Bessell et al. (1998) also concluded that model atmospheres are more reliable than the near-infrared absolute calibration measurements. However, Mégessier (1995) argued that the models were not reliable and that the near-infrared absolute calibration of Vega should be based on measurements that are independent of atmospheric models. Based on four model-independent measurements, Mégessier (1995) determined an averaged absolute flux density for Vega.

We directly compared the absolute calibrations of Vega determined by Cohen et al. (1992) and Mégessier (1995). We first used a third-order polynomial to fit the logarithm of the flux density for Vega as a function of the logarithm of the wavelength from Mégessier. The use of logarithms gives a nearly linear relationship. The wavelengths and flux densities for 0.0 mag were taken from Table 4 in Mégessier. We assumed a near-infrared magnitude of 0.02 mag for Vega, to be consistent with the visible magnitude assumed by Mégessier. We then interpolated to the wavelengths cited by Cohen et al. in their Table 1 to make direct comparisons to the Mégessier results.

The difference in the logarithm between the Mégessier and Cohen et al. absolute flux densities for Vega are shown in Figure 1. The mean of the difference is \(-0.0022 \pm 0.0031\). Thus, the absolute flux densities of Vega as given by Mégessier and Cohen et al. are indistinguishable. The uncertainty of the Mégessier and Cohen et al. flux densities for Vega are about 2% and 1.45%, respectively.

While the results discussed above show the consistency of the independent methods of Cohen et al. and Mégessier, we note the following caveats:

1. Both Cohen et al. (1992) and Mégessier (1995) rely on the absolute calibration of Vega at 0.5556 \(\mu\)m. Cohen et al. adopt a flux density of \(3.44 \times 10^{-8}\) W m\(^{-2}\) \(\mu\)m\(^{-1}\), while Mégessier adopts \(3.46 \times 10^{-8}\) W m\(^{-2}\) \(\mu\)m\(^{-1}\). Thus, Mégessier’s flux density for Vega is 0.6% higher at 0.5556 \(\mu\)m.

2. Recent work by Gulliver et al. (1994) and Peterson et al. (2004) indicate that Vega is pole-on and is a fast rotator. Thus, standard model atmospheres are not appropriate for Vega, as discussed by Bohlin & Gilliland (2004). Nonetheless, we have already shown the agreement between the Cohen et al. values and the model-independent results of Mégessier. In addition, Price et al. (2004) show that the Midcourse Space Experiment (MSX) absolute calibration experiment is in agreement with the Cohen et al. (1992) values for Vega, to within the experimental errors of 1%.

We show in Table 1 the flux densities for Vega for the isophotal wavelengths of the MKO-NIR filters. We have adopted the 1–5 \(\mu\)m flux densities for Vega as presented by Cohen et al. (1992) in their Table 1. We first computed the flux densities for Vega assuming a precipitable water vapor value of 2 mm at an air mass of 1.0, in addition to the parameters for Vega discussed in § 2.1. Since the model flux density for Vega is 1.9% lower than that used by Cohen et al. (1992), we increased our calculated values by 1.9%, and this is shown in Table 1. Bohlin & Gilliland (2004) also found that the flux density of Vega in the infrared was about 2% lower than that presented by Cohen et al. (1992).

For the V isophotal wavelength and flux density calculations, we used the absolute spectrophotometry for Vega given by Bohlin & Gilliland (2004). The V-filter profile used was that of Landolt (1992), obtained from Cohen et al. (2003; see the electronic version of the paper). The values we obtained for this V filter are shown in Table 1.

It is evident from the above discussion that there is no consistent published atmospheric model for Vega at both visible and infrared wavelengths. This is primarily because Vega is nearly pole-on and there is a range of temperature from the hotter pole regions to the cooler equatorial regions. A single-temperature model for Vega is therefore not realistic. The values of isophotal wavelengths and flux densities shown in Table 1 represent a best estimate based on absolute calibrations using blackbody sources, observations of standard stars, and atmospheric models.

We note that Mégessier (1995) found that the observed fluxes of Vega were about 2% higher than the atmospheric models. This problem was attributed to a possible near-infrared excess of Vega. However, Leggett et al. (1986) found no infrared excess from Vega compared to other A0 stars. This suggests that atmospheric models for Vega at near-infrared wavelengths are in error, possibly because Vega is observed pole-on.

### 2.3. Comment on the Definition of Zero Magnitude

Infrared photometric systems at 1–5 \(\mu\)m are usually defined as being based on the Johnson system or in a system in which the magnitude of Vega is taken to be 0.0. Examples of the former include systems at the University of Arizona (Campins et al. 1985), ESO (Wamsteker 1981), SAAO (Carter 1990),
AB magnitudes are defined as 0.02 or 0.03 mag. Examples of the latter include the systems CIT (Elias et al. 1982) and the Las Campanas Observatory (Persson et al. 1998). The UKIRT photometric system (Hawarden et al. 2001; Leggett et al. 2003) is based on the Elias et al. (1982) standard stars, so it follows the convention that the magnitude of Vega is 0.0 mag. Cohen et al. (1992) adopted a magnitude of 0.0 mag at infrared wavelengths, so that the flux density of Vega defines the flux density for 0.0 mag. This is continued in subsequent papers, and in Cohen et al. (2003) the nonzero magnitude of Vega at optical wavelengths is taken into account. Thus, when applying the results in Table 1, one must take into consideration which photometric system is used.

2.4. AB Magnitudes

The monochromatic AB magnitudes were defined by Oke & Gunn (1983) as

\[ AB = -2.5 \log f_\lambda - 48.60, \]  

(7)

where \( f_\lambda \) is in units of ergs cm\(^{-2}\) s\(^{-1}\) Hz\(^{-1}\) (see also Fukugita et al. 1996). The constant is set so that AB is equal to the V magnitude for a source with a flat spectral energy distribution. We adopt the Vega flux densities recommended by Bohlin & Gilliland (2004; their alpha_lyr_stis_002.fits file). The visible flux values are tied to a flux density of \( 3.46 \times 10^{-8} \) W m\(^{-2}\) μm\(^{-1}\) at 0.5556 μm, following Mégessier (1995). The isophotal wavelength at V is 5546 Å, and the isophotal flux density is \( 3.63 \times 10^{-20} \) ergs cm\(^{-2}\) Hz\(^{-1}\) (Table 1). A V magnitude of 0.026 from Bohlin & Gilliland (2004) is assumed. Then

\[ AB = -2.5 \log f_\lambda - 48.574, \]  

(8)

or for \( F_\lambda \) expressed in units of Jy, we have

\[ AB = -2.5 \log F_\lambda + 8.926. \]  

(9)

The AB magnitudes for Vega were calculated from equation (9) and are shown in the AB magnitude column of Table 1.

A great advantage of AB magnitudes is that the conversion to physical units at all wavelengths can be obtained with a single equation:

\[ F_\lambda = 3720 \times 10^{-0.4 AB}. \]  

(10)

The constant in equation (8) differs from that of Fukugita et al. (1996), because for Vega we assumed a different flux density value at V and adopted a different visual magnitude. However, it is within the uncertainty of the absolute calibration of 2% for Vega stated by Oke & Gunn (1983).

3. SUMMARY

1. Isophotal wavelengths, flux densities, and AB magnitudes for Vega are derived for the MKO-NIR filter set.

2. The absolute calibrations by Cohen et al. (1992) and Mégessier (1995) are shown to be identical within the uncertainties. We adopt the 1–5 μm absolute calibration of Cohen et al. to be consistent with the subsequent papers by Cohen and his colleagues.

3. The V-band isophotal wavelength and flux density is given for completeness using the renormalized Vega model and the absolute calibration adopted by Bohlin & Gilliland (2004). The constant in the AB magnitude definition was determined from the recent Hubble Space Telescope (HST) measurements of the V magnitude and flux density of Vega and differs from that defined by Oke & Gunn (1983).

4. There is no self-consistent atmospheric model for Vega at visible and infrared wavelengths. Improvements to Table 1 can be expected with models that take into account that Vega is observed pole-on, and also with observations of a grid of A0 stars (including Sirius) to eliminate the dependence on currently unreliable atmospheric models for Vega in the infrared.

We thank M. Cohen, D. Peterson, and T. Nagata for useful discussions and Steve Lord for making ATRAN available to us. A. T. T. was supported by NASA Cooperative Agreement number NCC 5-538. This research has made use of NASA’s Astrophysics Data System Bibliographic Services.

APPENDIX A

Other definitions for effective wavelengths are briefly discussed here. A more detailed discussion can be found in Golay (1974). In a manner similar to that used in equation (4), we can define the “effective wavelength” as

\[ \lambda_{\text{eff}} \int F(\lambda)S(\lambda) \, d\lambda = \int \lambda F(\lambda)S(\lambda) \, d\lambda, \]  

(A1)

and hence

\[ \lambda_{\text{eff}} = \frac{\int \lambda F(\lambda)S(\lambda) \, d\lambda}{\int F(\lambda)S(\lambda) \, d\lambda}. \]  

(A2)

This is the wavelength at which the flux distribution in energy units, integrated over the passband and then converted to
photon, equals the flux distribution in photon units integrated over the passband. Alternatively, \( F_\lambda \) \( S(\lambda) \) can be thought of as a probability distribution for detecting energy from the source, and therefore \( \lambda_{\text{eff}} \) is the mean wavelength of the passband as weighted by the energy distribution of the source over the band. In a similar manner, we can determine a mean wavelength for the passband as weighted by the photon distribution of the source over the band. In this case,

\[
\lambda_{\text{eff}} = \frac{\int \lambda F_\lambda(\lambda) S(\lambda) d\lambda}{\int \lambda F_\lambda(\lambda) d\lambda}.
\]  

(A3)

Here \( \lambda F_\lambda(\lambda) S(\lambda) \) can be thought of as the probability distribution for detecting a photon from the source. Note that both effective wavelength definitions depend on the spectral energy distribution of the source.

We can define a source-independent wavelength as follows:

\[
\int P(\lambda) S(\lambda) d\lambda = \int F_\lambda(\lambda) S(\lambda) d\lambda/\hbar \nu d\lambda,
\]  

(A4)

where \( P(\lambda) = F_\lambda(\lambda)/\hbar \nu \) is the photon flux from the object. Thus,

\[
\langle P_\lambda \rangle \int S(\lambda) d\lambda = \left\langle F_\lambda \right\rangle \int \lambda S(\lambda) d\lambda.
\]  

(A5)

Setting \( \langle P_\lambda \rangle = \lambda_0 \langle F_\lambda \rangle / \hbar c \) yields

\[
\lambda_0 = \frac{\int \lambda S(\lambda) d\lambda}{\int S(\lambda) d\lambda},
\]  

(A6)

which is the “mean wavelength” of the system.

We can express equation (1) in terms of frequency to derive another filter wavelength. From equation (1), we have

\[
N_\nu = \frac{1}{\hbar c} \int \lambda F_\lambda(\lambda) S(\lambda) d\lambda = \frac{1}{h} \int F_\lambda(\nu) S(\nu) d\nu/\nu.
\]  

(A7)

In a manner similar to the derivation given above for equation (A5),

\[
\frac{1}{c} \left\langle F_\lambda \right\rangle \int \lambda S(\lambda) d\lambda = \left\langle F_\lambda \right\rangle \frac{\int S(\nu)}{\nu} d\nu = \left\langle F_\lambda \right\rangle \int \frac{S(\lambda)}{\lambda} d\lambda,
\]  

(A8)

where the last equation results from the fact that \( d\nu = d\lambda/\lambda \). If we set

\[
\left\langle F_\lambda \right\rangle = \left\langle F_\lambda \right\rangle \lambda_{\text{pivot}}^2/c,
\]  

(A10)

we obtain

\[
\lambda_{\text{pivot}} = \sqrt{\frac{\int \lambda S(\lambda) d\lambda}{\int S(\lambda) d\lambda}}
\]  

(A11)

which is known as the “pivot wavelength” of the system. The pivot wavelength provides an exact relation between \( F_\lambda \) and \( F_\nu \) given by equation (A10). This definition is used in the HST Synphot Users Guide (Bushouse & Simon 1998; see also Koornneef et al. 1986).

The wavelengths based on the different definitions are shown in Table 2.

### REFERENCES


Table 2

<table>
<thead>
<tr>
<th>Filter</th>
<th>( \lambda_{\text{iso}} ) (( \mu \text{m} ))</th>
<th>( \lambda_{\text{eff}} ) (( \mu \text{m} ))</th>
<th>( \lambda_{\text{mean}} ) (( \mu \text{m} ))</th>
<th>( \lambda_{\text{pivot}} ) (( \mu \text{m} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>J</td>
<td>1.250</td>
<td>1.241</td>
<td>1.243</td>
<td>1.248</td>
</tr>
<tr>
<td>H</td>
<td>1.644</td>
<td>1.615</td>
<td>1.619</td>
<td>1.630</td>
</tr>
<tr>
<td>K</td>
<td>2.121</td>
<td>2.106</td>
<td>2.111</td>
<td>2.123</td>
</tr>
<tr>
<td>K</td>
<td>2.149</td>
<td>2.138</td>
<td>2.141</td>
<td>2.151</td>
</tr>
<tr>
<td>L</td>
<td>2.198</td>
<td>2.186</td>
<td>2.190</td>
<td>2.202</td>
</tr>
<tr>
<td>L</td>
<td>3.754</td>
<td>3.717</td>
<td>3.727</td>
<td>3.757</td>
</tr>
<tr>
<td>M</td>
<td>3.754</td>
<td>3.717</td>
<td>3.727</td>
<td>3.757</td>
</tr>
</tbody>
</table>