Solve the problems listed below, and write up your answers clearly and completely. Do not turn in rough work – instead, make a clean copy after checking your calculations. Use English sentences and phrases to explain your solution and describe key equations. Show your work!

1. In class we estimated the Sun’s central temperature $T_c$ using the ideal gas law. Another estimate of $T_c$ can be obtained by equating the thermal velocities of hydrogen nuclei with the velocities particles would have if they could orbit within the Sun without colliding with each other.

(a) Assume the Sun’s core has mass $M_{\text{core}} \simeq 2 \times 10^{29}$ kg and radius $R_{\text{core}} \simeq 7 \times 10^7$ m. What is the velocity $v_{\text{circ}}$ of a particle on a circular orbit at radius $r = R_{\text{core}}$?

(b) Suppose the particle in part (a) has the same mass as a hydrogen nucleus ($m_p \simeq 1.67 \times 10^{-27}$ kg). Set the particle’s kinetic energy to $\frac{3}{2} k_B T$, where $k_B \simeq 1.38 \times 10^{-23}$ kg m$^2$ s$^{-2}$ K$^{-1}$ is Boltzmann’s constant, solve for $T$, and compute it for the parameters given.

2. In an ideal gas, the average distance $\ell$ between gas particles is much greater than the particle radius $r$. Thus, the ratio $\ell/r$ is much greater than 1 for an ideal gas. In contrast, fluids have $\ell/r \simeq 1$.

(a) Hydrogen gas at STP has density $\rho = 0.090$ kg m$^{-3}$. Given that H$_2$ molecules have mass $m_{\text{H}_2} = 3.34 \times 10^{-27}$ kg and radii $r \simeq 1.2 \times 10^{-10}$ m, estimate $\ell/r$ in this gas.

(b) The plasma at the center of the Sun has density $\rho = 1.5 \times 10^5$ kg m$^{-3}$. For simplicity, assume this plasma is pure ionized Hydrogen (protons and electrons). Protons have mass $m_p = 1.67 \times 10^{-27}$ kg and radii $r \simeq 0.9 \times 10^{-15}$ m. Estimate $\ell/r$ for the protons in this plasma.

(c) Compare your answers to (a) and (b). Which material is a “more ideal” gas?

3. The fusion of 4 protons (p) to one $^4$He nucleus releases 26.7 MeV of energy (where 1 MeV $\simeq 1.60 \times 10^{-13}$ kg m$^2$ s$^{-2}$). Of this energy, $\sim 98\%$ heats the Sun’s interior, while $\sim 2\%$ is carried away by neutrinos.

(a) The Sun’s total light output is $L_\odot \simeq 3.8 \times 10^{26}$ kg m$^2$ s$^{-3}$. How many $4p \rightarrow ^4$He reactions take place in the Sun per second?

(b) How many neutrinos does the Sun emit per second?

(c) What is the average energy carried away by each neutrino?

(d) Estimate the number of solar neutrinos passing through your body every second.