1. This problem can be solved using only Kepler’s laws.

(a) The transfer orbit has

\[ r_p = 1 \text{ AU}, \quad r_a = 5 \text{ AU} \]

so: \[ a_+ = \frac{1}{2} (r_p + r_a) = 3 \text{ AU} \]

The empty focus (F) is 1 AU from Jupiter’s orbit, so the distance between the foci is \( 2ae = 4 \text{ AU} \), and the eccentricity is \( e = 4 \text{ AU} / 2a = 2/3 \). Then the semi-minor axis is

\[ b_+ = a_+ \sqrt{1 - e^2} = \sqrt{5} \text{ AU} = 2.24 \text{ AU} \]

(b) The orbital period is found using law III:

\[ P_+ = \sqrt{a_+^3 / (\text{AU}^3/\text{yr}^2)} = 3\sqrt{3} \text{ yr} = 5.20 \text{ yr} \]

It takes half an orbit to transfer, so the travel time is \( \frac{1}{2} P_+ = 2.60 \text{ yr} \).

(c) The area within the orbit is \( A_+ = \pi a_+ b_+ = 21.1 \text{ AU}^2 \). A line from the Sun to the ship sweeps out area at a constant rate

\[ \frac{dA}{dt} = \frac{A_+}{P_+} = 4.06 \text{ AU}^2 \text{ yr}^{-1} \]

Now, \( \frac{dA}{dt} = \frac{1}{2} r_p v_p = \frac{1}{2} r_a v_a \), so

\[ v_p = \frac{2}{r_p} \frac{dA}{dt} = 8.1 \text{ AU yr}^{-1} \approx 38.5 \text{ km/s} \]

\[ v_a = \frac{2}{r_a} \frac{dA}{dt} = 1.62 \text{ AU yr}^{-1} \approx 7.59 \text{ km/s} \]
1. (d) At $a = 1\text{ AU}$, a circular orbit has velocity $V_0 = 2\pi\text{ AU yr}^{-1}$, so the $\Delta V$ necessary to inject the ship into the transfer orbit is

$$
\Delta V = V_p - V_0 = 1.83 \text{ AU yr}^{-1} \approx 8.67 \text{ km/s}.
$$

2. (a) Since $\Theta$ is the only variable, the minimum $r$ value occurs when $\cos \Theta = 1$, and has the value

$$
\Gamma_p = \frac{(L/m)^2}{GM(1+e)}
$$

(b) At pericenter, the radius and velocity vectors are at right angles, so $L = m\Gamma_p V_p$. Insert this relation into the expression for $\Gamma_p$ found in part (a) and solve for $V_p$:

$$
\Gamma_p = \frac{(m r_p V_p/m)^2}{GM(1+e)} \Rightarrow V_p = \sqrt{\frac{GM(1+e)}{\Gamma_p}}
$$

(Note that for $e = 0$ we recover the circular orbit velocity.)

(c) The energy $E$ is everywhere equal to its value at pericenter:

$$
E = \frac{1}{2} m V_p^2 - \frac{G M m}{\Gamma_p} = \frac{1}{2} m \frac{G M (1+e)}{\Gamma_p} - \frac{G M m}{\Gamma_p}
$$

or

$$
E = -\frac{G M m}{2 \Gamma_p} (1-e)
$$

(d) The sign of $E$ depends on the sign of $1-e$:

- closed orbits ($e < 1$) have $E < 0$;
- parabolic orbits ($e = 1$) have $E = 0$;
- open orbits ($e > 1$) have $E > 0$.

(e) For $e = 2$, the energy $E > 0$, and at $r = \infty$ the $\frac{1}{r^2}$ term vanishes

$$
E = \frac{G M m}{2 \Gamma_p} = \frac{1}{2} m V_\infty^2 \Rightarrow V_\infty = \sqrt{GM/\Gamma_p}
$$

so the body escapes with a finite, nonzero velocity.