1. (a) The Dyson Sphere is straightforward; since all the radiation emitted by the star is re-radiated from the outer surface, we have

\[ L_* = L_{eb} = 4\pi R^2 \sigma_{SB} T^4 \]

So

\[ R = \sqrt{L_* / 4\pi \sigma_{SB} T^4} \]

\[ L_* = 3.8 \times 10^{26} \text{ Jy m}^{-2} \text{ s}^{-1} \quad T = 290 \text{ K} \]

\[ R \approx 2.75 \times 10^{10} \text{ m} \approx 1.83 \text{ AU} \]

(b) A Dyson Sphere would be a strong source of IR radiation. The peak of the black-body spectrum can be calculated using Wien's law:

\[ \lambda_{\text{max}} T = 2.9 \times 10^6 \text{ nm K}, \quad \text{so} \quad \lambda_{\text{max}} \approx \frac{2.9 \times 10^6 \text{ nm}}{T} \approx 10^4 \text{ nm} \]

for \( T = 290 \text{ K} \). This is in the mid-infared (\( \lambda_{\text{max}} \approx 10 \text{ micron} \)).

(c) To solve this problem you need the albedo of the rings. For simplicity, assume \( a = 0 \). There are two ways to find an answer: (i) work out the heating and cooling for the entire ring (which is very hard), or (ii) notice that any patch of surface area \( A \) has the same temperature as the whole. The second method is easier.

Assume the patch is facing the star at a distance \( R \).

The total energy input per unit time is

\[ W = A (1-a) F(R) = A \frac{L_*}{4\pi R^2} \quad \text{(for} \ a = 0) \]

The total energy output is

\[ L = 2A \sigma_{SB} T^4 \]

where the factor of 2 appears because both sides radiate.

(more)
1. (c) cont: Setting \( W = L \) and solving for \( R \):
\[
A \frac{L}{4\pi R^2} = 2 \pi \sigma_{SB} T^4
\]
\[
\Rightarrow R = \sqrt{\frac{L}{8 \pi \sigma_{SB} T^4}}
\]
Notice that \( A \) cancels out—it doesn't matter how big the patch was—and that this \( R \) is a factor of \( \frac{1}{2} \) smaller than part (a). That's because the patch rediscovers from both sides! For the given \( L = 1 \) and \( T \), we get
\[
R \approx 1.94 \times 10^3 \text{ m} \approx 1.29 \text{ AU}.
\]

2. (a) For planets in the solar system, the equilibrium temp. is
\[
T_p = 279 K \left(1 - a_p\right)^{\frac{1}{4}} \left(1 \text{AU}/r\right)^{\frac{1}{2}}
\]
Plugging in \( r = 30.1 \text{ AU} \), \( a_p = 0.31 \) yields
\[
T_p \approx 46 K
\]
(b) The critical temp. for molecules of mass \( m_m \) to escape is
\[
T_{cr} \approx \frac{1}{54} \frac{G M_p}{R_p} \frac{m_m}{k_B} \approx 1.2 \times 10^3 K
\]
where \( M_p = M_N \), \( R_p = R_N \), \( m_m = m_{H_2} \), and \( G \) and \( k_B \) are given in front of the book.

(c) Since \( T_p < 1.2 \times 10^3 K \), hydrogen will not escape from Neptune!

(d) Triton's equilibrium temp., due to its high albedo, is \( T_t \approx 32 K \). However, due to its smaller mass, the critical temp. for \( H_2 \) to escape is \( T_{cr} \approx 4.8 K \). Thus, Triton cannot retain \( H_2 \) molecules.

(more)
3. The m.f.p is
\[ l = \frac{1}{4\pi \Gamma_m^2 n} \]

(a) For \( \Gamma_m = \Gamma_{N_2} \approx 2 \times 10^{-10} \text{ m}, \) and \( n \approx 2.5 \times 10^{23} \text{ m}^{-3} \),
\[ l \approx 8 \times 10^{-8} \text{ m} \]

so a \( N_2 \) molecule can travel about 200 times its own diameter before having a collision.

(b) On the Moon, use \( \Gamma_m = \Gamma_{Ar} = 1 \times 10^{-10} \text{ m} \) and \( n \approx 2.5 \times 10^{10} \text{ m}^{-3} \) to get
\[ l \approx 3.2 \times 10^{-7} \text{ m} \]

This is \( \sim 10 \) times the Moon's radius, so the \( Ar \) atom travel ballistically (i.e., along conic sections).