Astronomy 242: Problem Set #2  
due February 1, 2016

_Solve the problems listed below, and write up your answers clearly and completely. Do not turn in rough work – instead, make a clean copy after checking your calculations. Use English sentences and phrases to explain your solution and describe key equations. Show your work!_

1. If you could compress the interstellar medium to the same density as air, it would resemble a thick black smoke. Verify this statement, assuming that it refers to the warm neutral medium, which has an number density \( n_{\text{WNM}} \simeq 4 \times 10^5 \text{ m}^{-3} \) and dims starlight by \( \sim 1 \text{ mag} \) per kpc (kiloparsec). For this problem, you may also assume that the WNM is a uniform mixture of hydrogen atoms and dust grains (ignoring helium), and that the compression process does not destroy the dust. The mass density of air at sea level is \( \rho_{\text{air}} \simeq 1.23 \text{ kg m}^{-3} \).

2. In a gas at temperature \( T \), collisions between gas particles establish a Maxwell–Boltzmann distribution,

\[
n(v)dv = n_0 \left( \frac{m}{2\pi k_B T} \right)^{3/2} e^{-\frac{1}{2}m v^2/k_B T} \frac{4\pi v^2 dv}{4\pi v^2},
\]

where \( n_0 \) is the number density of particles, and \( m \) is the particle mass. For a gas containing two or more types of particles, each type independently follows this distribution.

(a) What is the most probable particle speed, that is, for what speed does \( n(v) \) reach its maximum value?

(b) Using the parameters listed on p. 382 for the warm neutral medium (WNM), estimate the typical time between collisions. Assume the WNM is composed of pure atomic hydrogen, and that the radius of a hydrogen atom is \( R_H \simeq 5.3 \times 10^{-11} \text{ m} \).

(c) Compare this collision timescale to the half-life of hydrogen atoms in the spin-parallel state (see p. 382), which is \( \sim 1.1 \times 10^7 \text{ yr} \). Do most atoms in the WNM de-excite via collisions or via emission of radiation?

3. Now consider the warm ionized medium (WIM). In a plasma (ionized gas), particles need not physically collide like billiard balls; they merely need to come close enough for electrical forces to scatter them. The electrical interaction energy between two elementary charges \( e \) at separation \( r \) is

\[
U(r) = \frac{e^2}{4\pi \epsilon_0 r},
\]

where \( \epsilon_0 \) is permittivity of the vacuum. A pair of charged particles will be strongly scattered if they come close enough to make \( U(r) \) comparable to their combined kinetic energy.

(a) Using the parameters listed on p. 383, find the most probable velocities for protons (\( p \)) and electrons (\( e \)) in the WIM.

(b) Using these velocities, estimate the scattering radii for protons and electrons in the WIM. Compare your result to \( R_H \).