Solve the problems listed below, and write up your answers clearly and completely. Do not turn in rough work – instead, make a clean copy after checking your calculations. Use English sentences and phrases to explain your solution and describe key equations. Show your work!

1. The bolometric luminosity evolution of a simple stellar population of mass \(M_0\) and age \(\tau\) is approximately

\[
L_{\text{bol}}(\tau) \simeq \begin{cases} 
L_0, & \text{if } \tau < t_0, \\
L_0 \times (t_0/\tau)^{0.9}, & \text{if } \tau \geq t_0,
\end{cases}
\]

where \(t_0 \simeq 3 \times 10^6\) yr is the lifetime of a star at the high-mass end of the main sequence, and \(L_0 \simeq 1000 L_{\odot, \text{bol}} \times (M/M_{\odot})\).

(a) Consider a galaxy which formed its entire mass \(M_\ast = 10^{10} M_{\odot}\) of stars in a brief burst \(10^{10}\) yr ago. What is the present luminosity of this galaxy?

(b) Consider a galaxy which has been forming stars at a constant rate \(\dot{M}_\ast = 1 M_{\odot} \text{ yr}^{-1}\) for \(10^{10}\) yr. What is its present luminosity?

2. The broad H\(\alpha\) emission line of a typical Seyfert I galaxy has a width of \(\sim 160\) Å; in other words, the gas in the “broad-line region” is moving fast enough to doppler-shift the emitted light by \(\sim 80\) Å to either side of the normal wavelength of \(\lambda = 6563\) Å.

(a) How fast is the gas in the broad-line region moving?

(b) Can this motion be attributed to thermal velocities of individual gas atoms? Estimate the temperature necessary, and compare to the temperature at which hydrogen becomes ionized; what is your conclusion?

(c) If the gas in the broad-line region consists of clouds orbiting a super-massive black hole of mass \(M_\bullet = 10^8 M_{\odot}\), estimate the radius of the broad-line region.