Astronomy 242: Problem Set #6
due March 14, 2016

Solve the problems listed below, and write up your answers clearly and completely. Do not turn in rough work – instead, make a clean copy after checking your calculations. Use English sentences and phrases to explain your solution and describe key equations. Show your work!

1. A photon leaves a distant galaxy with comoving radius $r_o$ at time $t_e$ and is observed at time $t_o$. Assume that the universe is expanding, so between $t_e$ and $t_o$ the scale factor $a(t)$ increases with time. Let $r(t)$ be the distance to the galaxy at time $t$. Show that

$$r(t_e) < c(t_o - t_e) < r(t_o).$$

(Hint: recall that

$$r_o = \int_{t_e}^{t_o} \frac{dt}{a(t)} = \frac{r(t_e)}{a(t_e)} = \frac{r(t_o)}{a(t_o)}.$$ (2)

Also, if you can’t prove (1) is always true for an increasing $a(t)$, use the specific form $a(t) = (t/t_o)^{2/3}$ for a critical, matter-dominated universe.)

2. Consider an expanding universe with vanishingly small energy density: $u(t) = 0$ (in effect, this is the limiting case of an empty universe).

(a) Using the relativistic Friedmann equation, show that the only possible value for the curvature index is $\kappa = -1$. (Thus, all nearly-empty universes have hyperbolic geometry.)

(b) Show that the scale factor is a linear function of time: $a(t) = t/t_o$. 

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