Solve the problems listed below, and write up your answers clearly and completely. Do not turn in rough work – instead, make a clean copy after checking your calculations. Use English sentences and phrases to explain your solution and describe key equations. Show your work!

1. For some purposes, the core of the Sun can be treated as an independent, self-gravitating object. Assume the core is homogenous, and has mass \( M_c = 0.1 M_\odot \) and radius \( R_c = 0.1 R_\odot \). Then the core density and central pressure are

\[
\rho_c = \frac{3M_c}{4\pi R_c^3} \approx 1.4 \times 10^5 \text{ kg m}^{-3}, \quad P_c = \frac{3GM_c^2}{8\pi R_c^4} \approx 1.4 \times 10^{16} \text{ kg m}^{-1} \text{s}^{-2}.
\]

(a) Suppose that the core contains 50% hydrogen and 50% helium (by mass), both fully ionized. Given the core’s current density, how much degeneracy pressure could its electrons produce? Compare this pressure to the central pressure given above. Is degeneracy currently an important source of pressure in the Sun’s core?

(b) When the core’s hydrogen is completely used up, it will contract until it gets hot enough to ignite helium. Assume it contracts by a factor of 10, to radius \( R'_c = 0.01 R_\odot \). If the core remains homogenous, what density \( \rho'_c \) and central pressure \( P'_c \) will it have?

(c) At this stage the core is pure helium, fully ionized. What degeneracy pressure will the core’s electrons produce? Compare this pressure to the pressure \( P'_c \) you derived in (b). Is degeneracy pressure important at this point in the Sun’s evolution?

2. Supernova 2014J, produced by the thermonuclear explosion of a white dwarf, was discovered in the nearby spiral galaxy M82 earlier this decade. Spectroscopic measurements show the fireball is expanding at \( v_{\text{exp}} \approx 1.3 \times 10^7 \text{ m s}^{-1} \), or \( \sim 4.3\% \) of the speed of light.

(a) At maximum luminosity, this type of supernova has an absolute bolometric magnitude \( M_{\text{bol}} \approx -19.5 \) and a surface temperature \( T \approx 10,000 \text{ K} \). Assuming it radiates like a black-body, estimate the radius of the fireball when it reaches maximum luminosity.

(b) This type of supernova takes about two weeks to reach maximum luminosity. Use this fact and your result from part (a) to make an estimate of the expansion velocity which is independent of the spectroscopic data. Is your result consistent with the spectroscopic measurement?

(c) This type of supernovae is powered by thermonuclear burning of carbon to iron. Given that the white dwarf had a mass of \( M_{\text{Ch}} \approx 1.4 M_\odot \), and that burning this mass of carbon to iron yields an energy \( E = 2.8 \times 10^{44} \text{ kg m}^2 \text{s}^{-2} \), how fast would you expect the resulting fireball to expand? Is this consistent with the spectroscopic measurement and with your result for part (b)?