Lecture 18:
The structure of elliptical galaxies
Spheroidal systems with intrinsic axial ratios from about 1 to 0.5

Content: visible component is almost entirely stars (except for some of the largest ellipticals which contain hot X-ray emitting gas).

The dark/visible mass ratio is ~ 5-10

Should be simple? but they are actually fairly complex!
M87

the dominant elliptical galaxy in the Virgo cluster

note the globular clusters in its outer regions
Leo I - a dwarf elliptical in the Local Group
Gas content of elliptical galaxies

Elliptical galaxies are much more gas-rich than one usually thinks

<table>
<thead>
<tr>
<th>Type of gas</th>
<th>Temp K</th>
<th>Density cm⁻³</th>
<th>Mass M₀</th>
<th>Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hot</td>
<td>10⁷</td>
<td>0.1</td>
<td>10¹⁰</td>
<td>&gt;0.8</td>
</tr>
<tr>
<td>Warm</td>
<td>10⁴</td>
<td>10²</td>
<td>10⁴ - 10⁵</td>
<td>&gt;0.5</td>
</tr>
<tr>
<td>Cool</td>
<td>10²</td>
<td>10² - 10³</td>
<td>10⁷ - 10¹⁰</td>
<td>0.1 - 0.2</td>
</tr>
<tr>
<td>Dust</td>
<td>&lt;10²</td>
<td>&gt;10³</td>
<td>10⁵ - 10⁷</td>
<td>~0.8</td>
</tr>
</tbody>
</table>
Ellipticals in the X-ray

- Hot gas

Gas heated by SN explosions trapped in galaxy potential well.

Hot electrons produce X-rays by bremsstrahlung
Ellipticals:

- X-rays dominated by diffuse emission
- Hot gas distribution inhomogeneous (unlike optical)
- Some stirring mechanism in operation: possible periodic AGN activity
Optical properties of the ellipticals
Elliptical galaxies

Surface brightness of elliptical galaxies falls off smoothly with radius. Measured (for example) along the major axis of the galaxy, the profile is normally well represented by the $R^{1/4}$ or de Vaucouleurs law:

$$I(R) = I(0) e^{-kR^{1/4}}$$

where $k$ is a constant. This can be rewritten as:

$$I(R) = I_e e^{\left\{ -7.67 \left( \frac{R}{R_e} \right)^{0.25} - 1 \right\}}$$

where $R_e$ is the **effective radius** - the radius of the isophote containing half of the total luminosity. $I_e$ is the surface brightness at the effective radius. Typically, the effective radius of an elliptical galaxy is a few kpc.
Profile of elliptical galaxies can deviate from the $R^{1/4}$ law at both small and large radius.

Close to the center:
- Some galaxies have **cores** - region where the surface brightness flattens and is $\sim$ constant
- Other galaxies have **cusps** - surface brightness rises steeply as a power-law right to the center

A cuspy galaxy might appear to have a core if the very bright center is blurred out by atmospheric seeing.

HST essential to studies of galactic nuclei.
Results from HST

Graph showing surface brightness vs. mean radius (pc).

- 'Cores' (solid lines)
- 'Power-laws' (dashed lines)
To describe these observations, use profile suggested by the Nuker team:

\[ I(R) = I_b \left( \frac{R}{R_b} \right)^{-g} \left[ 1 + \left( \frac{R}{R_b} \right)^{g-\beta} \right]^{a} \left( \frac{R}{R_b} \right)^{-\gamma} \]

Represents a broken power law:

- Slope of \(-\gamma\) at small radii
- Slope of \(-\beta\) at large radius
- Transition between the two slopes at a break radius \(R_b\), at which point the surface brightness is \(I_b\)
- Remaining parameter \(\alpha\) controls how sharp the changeover is
γ = 0, 0.5, 1, 1.5

β = 1, 1.5, 2

Changing inner slope

Changing outer slope
Surveys of elliptical galaxies show that:

- The **most luminous** ellipticals have cores (typically a slope in the surface brightness distribution $\sim R^{-0.3}$ or flatter)
- **Low luminosity** ellipticals have power law cusps extending inward as far as can be seen
- At intermediate luminosities, mixture of cores and cusps

Unknown why elliptical galaxies split into two families…
In giant elliptical galaxies (cD galaxies) found at centers of some galaxy clusters, surface brightness at *large* radius may exceed that suggested by $R^{1/4}$ law:

- HST image of galaxy cluster Abell 2218.
- Lens (the cluster) is at a redshift $z \sim 0.3$
- Galaxies behind are at redshifts up to 5.58, and are strongly magnified and distorted:
What keeps elliptical galaxies in equilibrium?

mainly a balance between gravity and the pressure gradient associated with the random velocities (ie orbital motions) of the stars

The less massive ellipticals ($M < 10^{11} M_\odot$) are flattened by rotation

The more massive ellipticals ($M > 10^{11} M_\odot$) are flattened by their anisotropic velocity dispersion

Some of the faintest ellipticals like Leo I have very large dark/luminous mass fractions, up to about 50:1
Two important timescales

3) The **dynamical time** (rotation period, crossing time)
\[ \sim (G \rho)^{-1/2} \]
where \( \rho \) is a mean density. Typically
\[ 2 \times 10^8 \text{ yr} \] for galaxies.

2) The **relaxation time**. In a galaxy, each star moves in the potential field \( \Phi \) of all the other stars.

The density \( \rho(r) \) is the sum of \( 10^6 \) to \( 10^{12} \) \( \delta \)-functions.

As the star orbits, it feels the *smooth* potential of distant stars and the *fluctuating* potential of the nearby stars.

Are the fluctuations important?
Strong encounters

In a large stellar system, gravitational force at any point due to all the other stars is almost constant. Star traces out an orbit in the smooth potential of the whole cluster.

Orbit will be changed if the star passes very close to another star - define a strong encounter as one that leads to $\Delta v \sim v$.

Consider two stars, of mass $m$. Suppose that typically they have average speed $V$.

Kinetic energy: $\frac{1}{2} m V^2$

When separated by distance $r$, gravitational potential energy: $\frac{Gm^2}{r}$
By conservation of energy, we expect a large change in the (direction of) the final velocity if the change in potential energy at closest approach is as large as the initial kinetic energy:

$$\text{Strong encounter} \quad \frac{Gm^2}{r} \gg \frac{1}{2} mV^2 \Rightarrow r_s \sim \frac{2Gm}{V^2}$$

$\Rightarrow$ Strong encounter radius

Stars have random velocities $V \sim 300 \text{ km s}^{-1}$, which for a typical star of mass $0.5 \, M_{\odot}$ yields $r_s \sim 0.01 \text{ au}$.  

Strong encounters are very rare…
Time scale for strong encounters:

In time $t$, a strong encounter will occur if any other star intrudes on a cylinder of radius $r_s$ being swept out along the orbit.

Volume of cylinder: $pr_s^2 Vt$

For a stellar density $n$, mean number of encounters: $pr_s^2 Vtn$

Typical time scale between encounters:

$$t_s = \frac{1}{pr_s^2 Vn} = \frac{V^3}{4pG^2 m^2 n}$$

(substituting for the strong encounter radius $r_s$)

Note: more important for small velocities.
\[ t_s \gg 4 \cdot 10^{12} \left( \frac{V}{10 \ \text{km s}^{-1}} \right)^3 \left( \frac{m}{M_{\text{sun}}} \right)^{-2} \left( \frac{n}{1 \ \text{pc}^{-3}} \right)^{-1} \text{ yr} \]

Conclude:

stars in galaxies, never physically collide, and are extremely unlikely to pass close enough to deflect their orbits substantially.
Weak encounters

Stars with impact parameter $b >> r_s$ will also perturb the orbit. Path of the star will be deflected by a very small angle by any one encounter, but cumulative effect might be large.
Do these small cumulative fluctuations have a significant effect on the star’s orbit?

This is a classical problem - to evaluate the relaxation time $T_R$ - ie the time for encounters to affect significantly the orbit of a typical star.

Say $v$ is the typical random stellar velocity in the system

$m$ mass

$n$ number density of stars

Then

$$T_R = \frac{v^3}{8\pi G^2 m^2 n \ln \left( \frac{v^3 T_R}{2Gm} \right)}$$ (see B&T 187-190)

$$T_R/T_{dyn} \sim 0.1 \frac{N}{\ln N} \quad \text{where } N \text{ is the total number of stars in the system}$$
In galactic situations, $T_R$ is usually $\gg$ age

e.g. in the solar neighborhood, $m = 1$
   $n = 0.1 \text{ pc}^{-3}$
   $v = 20 \text{ km s}^{-1}$
   so $T_R = 5.10^{12} \text{ yr} \gg$ age of the universe

In the center of a large elliptical where $n = 10^4 \text{ pc}^{-3}$ and
$v = 200 \text{ km s}^{-1}$, $T_R = 5.10^{11} \text{ yr}$

(However, in the centers of globular clusters, the relaxation time
$T_R$ ranges from about $10^7$ to $5.10^9 \text{ yr}$, so encounters have a
slow but important effect on their dynamical evolution)

Conclusion: in elliptical galaxies, stellar encounters are negligible: are
collisionless stellar systems. $\Phi$ is the potential of the smoothed-out
mass distribution. (In spirals, encounters between disk stars and
giant molecular clouds do have some dynamical effect.)
Orbits and shapes
Dynamics of elliptical galaxies

Galaxies that appear elliptical on the sky may be intrinsically oblate, prolate, or triaxial, depending upon their symmetries:

- **Oblate**
- **Prolate**
- **Triaxial**

For an individual galaxy, can’t determine from an image what the intrinsic shape of the galaxy is.

Orbits of stars differ substantially in different types.
Orbits in elliptical galaxies

Classification of stellar orbits in elliptical galaxies is much more complicated than for disk galaxies. Most important distinction is between **axisymmetric galaxies** (prolate or oblate) and **triaxial galaxies**.

In an axisymmetric galaxy, there is a plane, perpendicular to the symmetry axis, in which gravitational force is central.

No azimuthal force, so component of angular momentum about symmetry axis (say, z-axis) is conserved

\[ L_z = m \left( r \dot{v}_z \right) \]

Only stars with \( L_z = 0 \) can reach center, other stars must avoid the center.
Orbits in elliptical galaxies

Triaxial potential: energy is conserved but not $L_z$

Simple example is the potential:

$$F(x) = \frac{1}{2} \left[ w_x^2 x^2 + w_y^2 y^2 + w_z^2 z^2 \right]$$

…which is the potential inside a uniform density ellipsoid. $\omega_x$ etc are constants. Star in this potential follows harmonic motion in each of the $x,y,z$ directions \textit{independently}.

Unless $\omega_x$, $\omega_y$ and $\omega_z$ are rational multiples of each other (eg 1:2:7) orbits never close - star completely fills in a rectangular volume of space in the galaxy.

Example is a \textbf{box orbit}. 
Orbits in elliptical galaxies

- Minor-axis tubes
- Box orbits
- Outer major-axis tubes
- Inner major-axis tubes
- Boxlets

Shape of tubes only in triaxial potential
Fine structure in elliptical galaxies

Contours of constant surface brightness often depart slightly from true ellipses.

_ Twists: _ major axis of the isophotes changes angle going from the center of the galaxy to the edge. This can be interpreted as a projection effect of a triaxial galaxy in which the ellipticity changes with radius:

View ellipses on flat surface from lower left

See apparent twist in the isophotes
Fine structure in elliptical galaxies

Surface brightness distribution can also depart slightly from ellipses:

- **Boxy isophotes**
- **Disky isophotes**

Normally subtle distortions
Luminous ellipticals are often boxy, midsized ellipticals disky

Could classify ellipticals based on their degree of boxiness / disky-ness - S0s would then be continuation of a trend to increasing diskiness.
Faber-Jackson relation

Analog of the Tully-Fisher relation for spiral galaxies. Instead of the peak rotation speed $V_{\text{max}}$, measure the velocity dispersion along the line of sight $s$. This is correlated with the total luminosity:

$$L_V = 2 \times 10^{10} \left( \frac{s}{200 \ \text{kms}^{-1}} \right)^4 L_{\text{sun}}$$

Can be used as a (not very precise) distance indicator.
Fundamental plane

Recall that for an elliptical galaxy we can define an effective radius \( R_e \) - radius of a circle which contains half of the total light in the galaxy. Measure three apparently independent properties;

- The effective radius \( R_e \)
- The central velocity dispersion \( \sigma \)
- The surface brightness at the effective radius \( I_e = I(R_e) \)

Plot these quantities in three dimensions - find that the points all lie close to a single plane!

Called the fundamental plane.
Fundamental plane

Plots show edge-on views of the fundamental plane for observed elliptical galaxies in a galaxy cluster.

Approximately:

\[ R_e \mu S^{1.24} I_e^{-0.82} \]

Measure the quantities on the right hand side, then compare apparent size with \( R_e \) to get distance

Origin of the fundamental plane unknown...
Evolution of the Fundamental Plane (Treu et al 2005)

142 spheroidals: HST-derived scale lengths, Keck dispersions

Increased scatter/deviant trends for lower mass systems?

If \( \log R_E = a \log s + b S_B + \gamma \)

Effective mass \( M_E \propto \sigma^2 R_E / G \)

So for fixed slope, change in FP intercept \( \Delta \gamma \propto \Delta \log (M/L)^i \)
Evolution of the Intercept $\gamma$ of the FP

Strong trend: lower mass systems more scatter/recent assembly
Measuring distances to the ellipticals:

- can use Faber-Jackson
- but more interesting method is Tonry's surface brightness fluctuation method
Surface brightness fluctuations

Can use the fact that elliptical galaxies are made of stars to devise a cunning method for measuring distances.

Suppose we observe the same region of two identical elliptical galaxies at different distances:

- **Galaxy 1:** distance $D$
  - Number of stars which contribute to light in one pixel is $N$

- **Galaxy 2:** distance $2D$
  - Number of stars $4N$
Surface brightness fluctuations

Since galaxy 2 is twice as far away, its 4N stars contribute same average flux into the pixel as galaxy 1’s N stars (inverse square law -> constancy of surface brightness).

But suppose we observe many pixels at the same radius in the galaxies. How much variation in the flux do we expect due to fluctuations in the number of stars?

\[
\begin{array}{ccc}
I & \Delta I & \Delta I / I \\
Galaxy 1: & N & N^{1/2} & N^{-1/2} \\
Galaxy 2: & 4N & 2N^{1/2} & 0.5 N^{-1/2}
\end{array}
\]

Fluctuations in surface brightness scale as 1 / D
Surface brightness fluctuations

Use this as a distance indicator:

• Take image of elliptical galaxy
• Measure surface brightness fluctuations (subtract a `smoothed’ image of the galaxy, look at what’s left)
• Compare to the fluctuations of a galaxy of known distance

Perhaps the most elegant method:
• Only requires imaging
• Based just on statistics!

Many practical difficulties… dust, individual bright stars etc. but generally good method.
Elliptical formation and the present day properties
“Merger hypothesis”: ellipticals form by major mergers of … but what merged when?

Do present day disk mergers evolve into elliptical galaxies?
Making ellipticals by mergers of disk galaxies
Collisionless mergers of disks

Encounter survey: statistical properties of 96
Collisionless mergers of disks

galaxies: formation of ellipticals?! (Toomre & Toomre 1972,1977)

- Equal-mass remnants are spherical, slowly rotating, show shells, loops etc. similar to elliptical galaxies (Barnes 1988,1992; Hernquist 1992,1993)

- Phase-space constraints and kinematics rule out pure exponential disks as progenitors for ellipticals - bulges and/or dissipation needed (Hernquist 1993, Jesseit et al. 2005, Naab & Trujillo 2006, Cox et al. 2006)


- Shape of the LOSVD in general not in agreement with observed rotating ellipticals (Bendo & Barnes 2000; Naab & Burkert 2003; Gonzalez-Garcia, Cesar & Balcells 2005; Jesseit, Naab & Burkert 2005)
Problem....

Present day ellipticals are very uniform and uniformly old.
An Elliptical spectrum
But Ellipticals have uniformly old populations
Implying uniform early-type galaxy formation
Elliptical formation caught in the act?

- Perhaps the submillimeter galaxies are the ellipticals in formation?
- $z \sim 2–6$
- Massive objects
  - $\sim 10^{11} \, M$
- High star formation rates
  - $\sim 1000 \, M/yr$

But need to switch off subsequent star formation and have subsequent mergers be dry....
Summary:

Ellipticals are more complex than they look.
Formation processes are still not fully understood: again we don’t have a full explanation for the fundamental plane or the uniform stellar populations.
End