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Short Communication

Buffering of Sublimation Loss of Subsurface Ice by Percolating Snowmelt: A Theoretical Analysis

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ABSTRACT

Subsurface ice in cold hyperarid conditions retreats by sublimation and diffusion through the overlying soil layer. Here, it is shown that percolating meltwater, if present, can counterbalance sublimation loss effectively and thus increase the persistence time of subsurface ice. Time averaging of transport equations is used to evaluate the significance of percolation in an otherwise complex dynamical system. The reduction in sublimation loss is approximately given by the amount of meltwater multiplied by the percolation depth and divided by the depth to the ice table. It is plausible that percolation is even more effective during a warmer, wetter climate. Copyright © 2009 John Wiley & Sons, Ltd.

KEY WORDS: Antarctic Dry Valleys; mathematical geophysics; permafrost degradation; sublimation; ground ice

INTRODUCTION

Subsurface ice in hyperarid permafrost conditions, such as in non-glaciated areas of Antarctica, retreats by sublimation and diffusion through the overlying soil layer. More and more examples can be found in the literature where, at least superficially, subsurface ice is found to be older than expected based on theoretical recession rates (Woodcock, 1974; Sugden *et al*., 1995; Schäfer *et al*., 2000). Unfortunately, the physical conditions at the field sites are incompletely known. For proper theoretical modelling, long time-series of surface temperature, humidity and snow cover would be necessary, along with physical properties of the soil layer overlying the ice. In the absence of more field data, it is reasonable to look towards theory for insight. Here it is shown that percolating water can be very effective in reducing sublimation loss.

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While no profound contradiction between the emplacement age of subsurface ice and proper theoretical calculations has been clearly established (e.g., Ng *et al*., 2005), we would like to identify potential mechanisms that can (or could) maintain ancient ice near the surface over long times. The following mechanisms for reducing recession rates, compared to sublimation into a dry atmosphere and beneath a perennially dry surface, can be considered.

1. Atmospheric humidity: Even in the absence of any seasonal snow or frost on the surface, atmospheric humidity can balance the vapour pressure of the subsurface ice. For example, at the location of the meteorological station in Beacon Valley, Antarctica, it has been shown that atmospheric vapour slows the sublimation loss by a factor of three (Schorghofer, 2005).
2. Recurring snow cover: Any temporary snow or frost cover is a source of surface humidity that reduces sublimation loss, as pointed out by Kowalewski *et al*. (2006), Hagedorn *et al*. (2007), McKay (2008) and others.

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3. Meltwater: Occasional snow or frost may melt and percolate before the water returns to the atmosphere. Even if meltwater does not infiltrate deep enough to become part of the perennial ice, it acts as a buffer between the subsurface ice and the atmosphere. It is this effect that is discussed here.

All three mechanisms are ultimately based on a vapour density increase on or near the surface that counteracts the sublimation loss from the ice table. The ice table is said to be in equilibrium if any water vapour it loses is resupplied from above at the same time or close to the same time, such that there is no net loss to the atmosphere and the ice table does not recede at all. Climate conditions can be on either side of this equilibrium. If (at any depth) ice is stable but voids remain, interstitial ice will accumulate from vapour or liquid derived from the surface. Porous soils can therefore be replenished with interstitial ice. On the other hand, replenishment of a layer that consists predominately of ice (such as a glacial remnant) is probably impossible after the overlying sublimation till has formed. The ice table only remains near the surface during a prolonged period if the retreat rate is small.

TIME-AVERAGED TRANSPORT EQUATION WITH PERCOLATION

The model considered here is one-dimensional; it does not include lateral movement of water. Transport takes place between the surface and the ice table at depth z_t. The use of time-averaging of transport equations to calculate long-term loss rates is described in Schorghofer (2007), but has been implicitly used elsewhere. Here this analysis is extended to include the liquid phase. The model includes four phases of H_2O: vapour (subscript v), liquid (subscript ℓ), solid and adsorbed or hydrated water. Transport occurs only in the vapour and liquid phase.

The total downward transport of H_2O is

$$J = -D\frac{\partial \rho_v}{\partial z} + J_\ell \quad \text{for} \quad 0 < z < z_t \tag{1}$$

where D is the vapour diffusivity of the soil, ρ_v the (mass) density of water vapour, J_ℓ the downward flux of liquid water and z the depth coordinate (positive downward) with the surface at $z = 0$. The vapour density in the pore volume, ρ_v, is related to the vapour density relative to total volume, $\overline{\rho}_v$, by a factor of porosity. Local conservation of mass requires

$$\frac{\partial}{\partial t}(\overline{\rho}_v + \overline{\rho}_{\text{ice}} + \overline{\rho}_{\text{ads}} + \overline{\rho}_\ell) + \frac{\partial J}{\partial z} = 0 \quad \text{for} \quad 0 < z < z_t \tag{2}$$

where overbars indicate that the density is relative to the total volume and t represents time.

The key is to average over periodic variations by integrating over recurrence intervals of duration P. If the recurrence is seasonal, then P is one year. These time averages are indicated with angle brackets i.e. $\langle\rangle$. The ice table is nevertheless permitted to retreat gradually. This mathematical procedure distinguishes between fast (periodic) oscillations and slow (secular) changes. The averaging is justified if the change in ice table depth over a time P has a negligible influence on the temperature profile and the percolation behaviour.

The average of the time derivative in Equation (2) is zero for each of the four contributions, because each density returns to the same value after every recurrence interval. Hence $\partial\langle J\rangle/\partial z = 0$, which implies that the total H_2O flux is constant with depth i.e. from the surface to the ice table. By averaging Equation (1),

$$\langle J\rangle = -D\frac{\partial\langle\rho_v\rangle}{\partial z} + \langle J_\ell\rangle \tag{3}$$

where $\langle J\rangle$ is (minus) the net loss rate. Upon integration, this leads to

$$-D\langle\rho_v\rangle + \int_0^z \langle J_\ell\rangle dz = \langle J\rangle z + c. \tag{4}$$

At the surface where $z = 0$,

$$-D\langle\rho_v(0)\rangle = c \tag{5}$$

which sets the integration constant c. At the lower boundary (the ice table at depth z_t), Equation (4) becomes

$$-D\langle\rho_v(z_t)\rangle + \int_0^{z_t} \langle J_\ell\rangle dz = \langle J\rangle z_t + c. \tag{6}$$

Hence,

$$\langle J\rangle = -D\frac{\langle\rho_v(z_t)\rangle - \langle\rho_v(0)\rangle}{z_t} + \frac{1}{z_t}\int_0^{z_t}\langle J_\ell\rangle dz. \tag{7}$$

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Let $\mathcal{J}$ denote the flux we would have calculated without percolation of the liquid, then

$$\langle \mathcal{J} \rangle = -D \frac{\langle \rho_v(z_t) \rangle - \langle \rho_v(0) \rangle}{z_t}. \quad (8)$$

This leads to the final result for the buffering by percolation:

$$\frac{1}{z_t} \int_0^{z_t} \langle J_\ell \rangle dz = \langle J \rangle - \langle \mathcal{J} \rangle. \quad (9)$$

We now see that the effect of downward percolation of liquid water on net loss is fully determined by $\langle J_\ell \rangle$, which depends on depth but not on timing. Equation (9) is of great practical value, because it specifies that only the time-averaged quantity $\langle J_\ell \rangle(z)$ needs to be determined in field measurements. The net loss can then be obtained from Equations (8) and (9). The time-integrated J_ℓ, which is simply related to $\langle J_\ell \rangle$ by a factor of P, is the amount of water that traverses a given horizontal cross-section per recurrence interval. For example, if a drop of water moves from the surface to a particular depth, it does not matter how fast or when it moves; only its final depth matters.

Without the liquid phase, the vapour flux through the soil layer is determined in the long run by the vapour density on the surface and at the ice table, as in Equation (8). Water that percolates downwards and ultimately returns to the atmosphere in the form of vapour reduces this limited transport capacity for vapour. The net loss is reduced by the column-averaged (downward) flux of liquid water. (By definition, ice loss corresponds to $\langle J \rangle < 0$.)

Gravity is not the only force that can cause transport of liquid water. Migration to the freezing front and capillary action can even move water upwards. If water moves only downwards, then $J_\ell \geq 0$ everywhere and anytime. In this case, the left-hand side in Equation (9) is certainly non-negative and the buffering is bounded from below by zero, which simply means that any percolation reduces sublimation loss.

When a snow mass m_ℓ per area and time melts on the surface, then $\langle J_\ell(0) \rangle = m_\ell$. The water might: percolate and evaporate; percolate, freeze and sublimate; freeze and thaw to percolate further; or, after it percolated downwards, some of it might even move upwards in the liquid phase. As less water is expected to reach greater depths, J_ℓ decreases with depth, i.e.

$$\langle J_\ell(z) \rangle \leq \langle J_\ell(0) \rangle = m_\ell. \quad (10)$$

(In principle, there can be another contribution to J_ℓ from melting of ice that accumulated in the dry subsurface by diffusion rather than by percolation. Here, we only consider the contribution of melting on the surface.) Further, denote the maximum depth to which the liquid infiltrates by z_p. In the field, z_p may be recognised as either the thickness of the active layer or the depth at which peak temperature never exceeds the melting point.

With the inequality from Equation (10), the integral in Equation (9) becomes

$$\int_0^{z_t} \langle J_\ell \rangle dz = \int_0^{z_p} \langle J_\ell \rangle dz \leq \int_0^{z_p} m_\ell dz = m_\ell z_p. \quad (11)$$

Summarising the inequalities,

$$\langle J \rangle - \langle \mathcal{J} \rangle \leq m_\ell \frac{z_p}{z_t} \leq m_\ell. \quad (12)$$

Hence, the reduction of the net loss of ice by percolating meltwater is bounded from above by $m_\ell z_p / z_t$. This result suggests the buffering by melt can be remarkably large, namely of the same order of magnitude as the amount that melts. The comparison becomes intuitive when the mass fluxes m_ℓ and $\langle \mathcal{J} \rangle$ are both expressed in terms of velocities. We can think of them as the thickness of a layer that melts and the distance the ice table retreats, respectively, over the same time period.

Further, it can be argued that this is not only an upper bound, but an order of magnitude estimate as long as most of the water percolates a distance not dramatically smaller than z_p and little or no water moves upwards in the liquid phase. For example, if the ice table is at 50 cm depth, and all meltwater percolates to 10 cm depth before it disappears, then the net buffering is exactly a factor of five less than m_ℓ.

This can be demonstrated more formally by example. Suppose the percolating meltwater reaches uniform density from depth 0 to z_p. Its density is then determined by $\overline{\rho}_\ell z_p = m_\ell P$. We can avoid specifying much about the time evolution, but imagine that the water reaches a uniform density before it returns to the atmosphere. The total mass per area that traversed the horizontal cross-section at depth z over a period P is

$$\int_0^P J_\ell dt = \overline{\rho}_\ell (z_p - z) \quad \text{for} \quad z < z_p \quad (13)$$

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and zero for $z_p < z < z_t$. From this, the time-averaged flux is

$$\langle J_\ell \rangle = m_\ell (1 - z/z_p) \quad \text{for} \quad z < z_p. \tag{14}$$

The relevant integral becomes

$$\int_0^{z_t} \langle J_\ell \rangle dz = m_\ell \int_0^{z_p} \left(1 - \frac{z}{z_p}\right) dz = m_\ell \frac{z_p}{2} \tag{15}$$

and the reduction in loss rate by the percolating liquid is therefore, according to Equation (9),

$$\langle J \rangle - \langle \mathcal{J} \rangle = m_\ell \frac{z_p}{2z_t}. \tag{16}$$

Comparison with Equation (12) demonstrates that the buffering is of order $O(m_\ell z_p / z_t)$.

At study sites in Beacon Valley, Antarctica, theoretical sublimation loss rates are of the order one-tenth of a millimetre per year (Hindmarsh *et al*., 1998; Schorghofer, 2005; Kowalewski *et al*., 2006). Even minute amounts of recurring melting can therefore play a dominant role. For example, melting of a 1 mm thick layer per year that percolates 5 cm deep would be approximately sufficient to suppress the sublimation loss entirely. No time series of snow cover or meltwater is available in Beacon Valley, but soil surface and air temperature have been measured by a meteorological station (Doran *et al*., 1995) since November 2000. In the seven years from 2001 to 2007, the soil surface temperature was above freezing 7.5% of the time. Even at 10 cm depth, temperature was above freezing 1.3% of the time. This means that the percolation could be at least this deep, although the active layer in the upper Beacon Valley is known to contain little moisture (Marchant and Head, 2007).

In summary, it is possible to use basic arguments of time averaging and mass conservation to show that liquid percolation reduces sublimation loss as quantified in Equation (9); the loss reduction is bounded from above as shown in Equation (12) and is often not much smaller than this upper bound. Periodic supply of a small volume of meltwater may be able to protect ancient ice from rapid sublimation. Since percolating water is so efficient in reducing sublimation loss, quantification of this process with field data would be particularly promising. Percolation is unlikely to be spatially uniform, and a corresponding inhomogeneity in the depth of the ice table may be a hallmark of percolation-reduced loss rates.

DISCUSSION: RESPONSE TO CLIMATE WARMING

An accurate explanation for the persistence of ancient ice should work not only in the current climate but also for a broader range of climate conditions, and especially during warm periods when sublimation from the ice table is rapid. Here, we speculate about the relative importance of percolation during a relatively warm and wet climate period. There are two competing responses in the model: an increase of the vapour pressure of the ice table and an increase in snowmelt.

The former contribution is given by Equation (8). The saturation vapour density increase in response to a small but persistent temperature increase ΔT can be derived from the Clausius–Clapeyron equation

$$\frac{\rho_v(T + \Delta T)}{\rho_v(T)} \approx 1 + \frac{\Delta T}{T}\left(\frac{H}{RT} - 1\right), \tag{17}$$

where H is the sublimation enthalpy, R the universal gas constant and $H/(RT) \approx 24$. The vapour density at the ice table therefore increases by a factor of $1 + \Delta T / 11^\circ\text{C}$. The sublimation loss $\mathcal{J}$ in Equation (8) also depends on the humidity on the surface, $\langle \rho_v(0) \rangle$. However, if $\langle \rho_v(0) \rangle$ is comparable to $\langle \rho_v(z_t) \rangle$, then the ice is already stabilised such that we are only interested in the case where $\langle \rho_v(0) \rangle$ is small and $\mathcal{J}$ is approximately proportional to $\langle \rho_v(z_t) \rangle$. Hence, the relative change in $\mathcal{J}$ is approximately given by Equation (17).

For a given snow cover, the melt volume will increase depending on the heat available on positive-degree days. Suppose the surface temperature has a Gaussian distribution $p(T;T_m)$ with mean temperature T_m and standard deviation σ. Since melting is rare, it can be assumed that $\sigma \ll 0^\circ\text{C} - T_m$. After asymptotic expansion of the Error function in terms of an exponential, we obtain for the fraction of time above melting:

$$\int_{0^\circ\text{C}}^{\infty} p(T;T_m) dT \approx \frac{\sigma}{\sqrt{2\pi}(0^\circ\text{C} - T_m)} e^{-\frac{(0^\circ\text{C} - T_m)^2}{2\sigma^2}}. \tag{18}$$

If all temperatures increase by ΔT, this time fraction increases by a factor of

$$\frac{\int_{0^\circ\text{C}}^{\infty} p(T;T_m + \Delta T) dT}{\int_{0^\circ\text{C}}^{\infty} p(T;T_m) dT} \approx 1 + \Delta T \frac{0^\circ\text{C} - T_m}{\sigma^2}. \tag{19}$$

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The heat available for melting involves a prefactor of (T – 0°C). After some calculation,

$$\frac{\int_{0^\circ C}^{\infty}(T+\Delta T-0^\circ C)p(T;T_m+\Delta T)dT}{\int_{0^\circ C}^{\infty}(T-0^\circ C)p(T;T_m)dT} \approx 1+2\Delta T\frac{0^\circ C - T_m}{\sigma^2}. \quad (20)$$

For $T_m = -21^\circ C$ and $\sigma = 13^\circ C$, representative of upper Beacon Valley, Equation (18) yields temperatures above freezing about 7% of the time. The factor in Equation (20) is $1+\Delta T/4^\circ C$. This variable therefore increases faster than the sublimation loss. Both are extremely sensitive to ΔT.

Snow accumulation depends on factors external to the site and cannot be estimated within this model, but warming may be accompanied by increased snowfall. These thoughts about climate response are rather idealised but they suggest that, in a warmer climate, increased snowmelt more than compensates the increase in vapour pressure of the ice. The ice could therefore readily persist through such periods. Alternatively, percolation would become insignificant during a colder drier climate period when sublimation loss is, in any case, slow.

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